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AN ESSAY
ON
THE TEETH OF WHEELS.



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AN ESSAY

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COMPREHENDING
PRINCIPLES, AND THEIR APPLICATION
IN PRACTICE,
TO
MILLWORK
AND OTHER MACHINERY.
WITH NUMEROUS FIGURES.

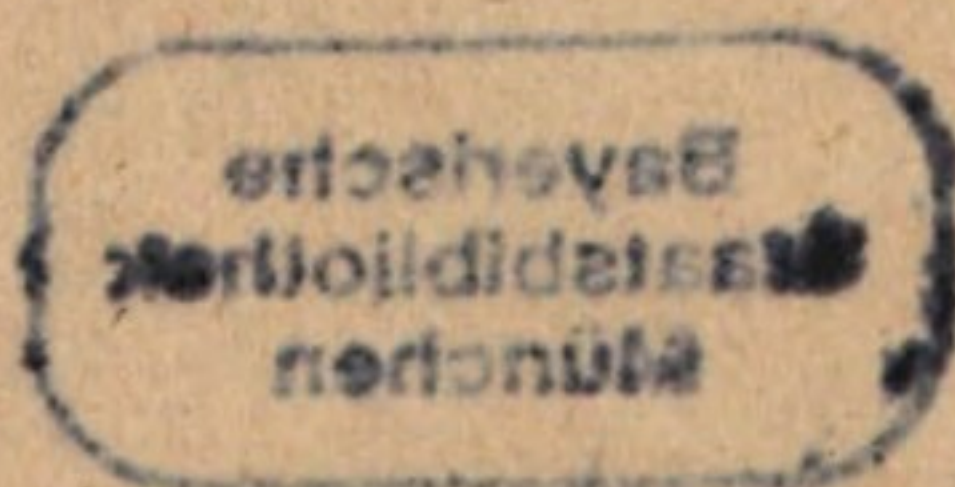
BY ROBERTSON BUCHANAN,
ENGINEER.

REVISED
BY PETER NICHOLSON, ARCHITECT, &c.

LONDON :

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1808.



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ILLUSTRATIVE OF THE THEORY

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PREFACE.

LED from situation, as well as curiosity, to attend very minutely to some parts of practical mechanics, one of the objects, which early attracted the notice of the author of the following short Essay, was the figure of the Teeth of Wheels. He observed, that, in forming these teeth, workmen followed rules for which they could assign no satisfactory reason. Nor did he then find in books the information he wanted: the subject seemed to him to require a detail and simplification, which no English writer, with whom he was acquainted, had given it.—After-

wards, indeed, he found that some French mathematicians had treated it with much attention. But their works, though sufficiently clear to those who have studied mathematics, are too abstract to be of general utility. In the following Essay, therefore, such an elucidation of the subject has been attempted, as might render it plain to the operative mechanic—an object, which will appear the more important, the more we consider the great variety of useful purposes to which wheel-work is applied.

De La Hire and Camus, are the two French writers, who have treated most extensively this branch of mechanics.—From the work of the latter, who has written more accurately, and more fully, the author has borrowed largely; nor has he scrupled to take from others,

whatever he found to suit his purpose, and to make the fullest use of the communications of his friends.

Of the method followed, it will be sufficient to remark, that the subject naturally suggested these two general divisions—First, the Principles of the Configuration of the Teeth of Wheels:—Secondly, the application of these to practice.

The first chapter contains the Principles—The second, their Application, with certain modifications—1st to *Spur Geer*, under which are comprehended, the *Wheel* and *Trundle*; the *Wheel* and *Pinion*; the *internal Pinion*, and the *Rack* and *Pinion*.—And, 2dly, to *Bevel Geer*.

A third chapter is added, which con-

tains a manner of forming *Spur Wheels*, upon principles somewhat different from those considered in the preceding chapter.

In the following pages, no pretensions are made, either to invention or profound investigation. The writer has studied perspicuity alone, and will have completely attained his object, if he has only been fortunate enough to give such a view of the various kinds of teeth, as will enable the artist to form some judgment of their respective merits, and to execute any of them with accuracy and ease. For this purpose it has been his aim to divest every part of the subject of obscurity, and to accommodate it to those who possess not the advantages of a mathematical education. But he is far from saying, that they will not find some difficulties, particularly in the first chap-

ter ; nor will they, perhaps, fully understand the truths it contains, till they see their relation to practice pointed out in the second.—He found, that without becoming exceedingly prolix, there was no avoiding the use of some mathematical terms, but of these he has given definitions, either as the terms themselves occur, or at the conclusion of the Essay*.

* This Preface was written several years before the translation of Camus was published.

1 Union Place, Glasgow,
February, 1807.

CONTENTS OF ESSAY I.

ON THE

TEETH OF WHEELS.

	PAGE
Preface,	v
General Definitions,	11
CHAP. I.—Of the Principles of the Configuration of the Teeth of Wheels,	15
Proposition,	18
Definitions,	20
Corollaries,	22
CHAP. II.—Of the Application of the Principles of the Configuration of the Teeth of Wheels,	36
SECT. I.—Of Spur Geers,	ib.
Of the Wheel and Trundle,	37
Of the Wheel and Pinion,	45
Remarks,	48
Rule to determine the length of the Teeth of Wheels,	72
Observations,	73
Of the Internal Pinion,	74
Of the Back and Pinion,	78
SECT. II.—Of the Bevel Geer,	81
CHAP. III.	92
Supplementary Observations,	96
Remarks on the Friction of Wheel Work, and on the Forms best suit-	

	PAGE
ed for Teeth—(In a Letter from Dr Young),	101
Supplementary Definitions,	111
Postscript,	118
APPENDIX—A Practical Inquiry respecting the the Strength and Durability of Teeth of Wheels used in Millwork,	121
General Observations on the Wheel- work of Mills,	122
Horse's Power,	130
Force of Men,	133
Explanation of the Table of Wheels in actual use in Mill-work,	135
Observations on the Table of Wheels in actual use in Mill-work, . . .	137
Tables of Pitches,	141—147
Table of Pitches of Wheels,	152
Practical Observations with regard to making of Patterns of Cast-iron Wheels,	153
Materials for Patterns,	160
Table of the Radii of Wheels, from Ten to Three Thousand Teeth, .	162
APPENDIX, No. II. —On the Use of Charts, and some further Explanation of the Construction of the Tables of Pitches of Wheel-work,	165
Letter from Mr James Carmichael, of Dundee, respecting the strength, &c. of Wheel-work,	176
Errata,	179
Index,	181

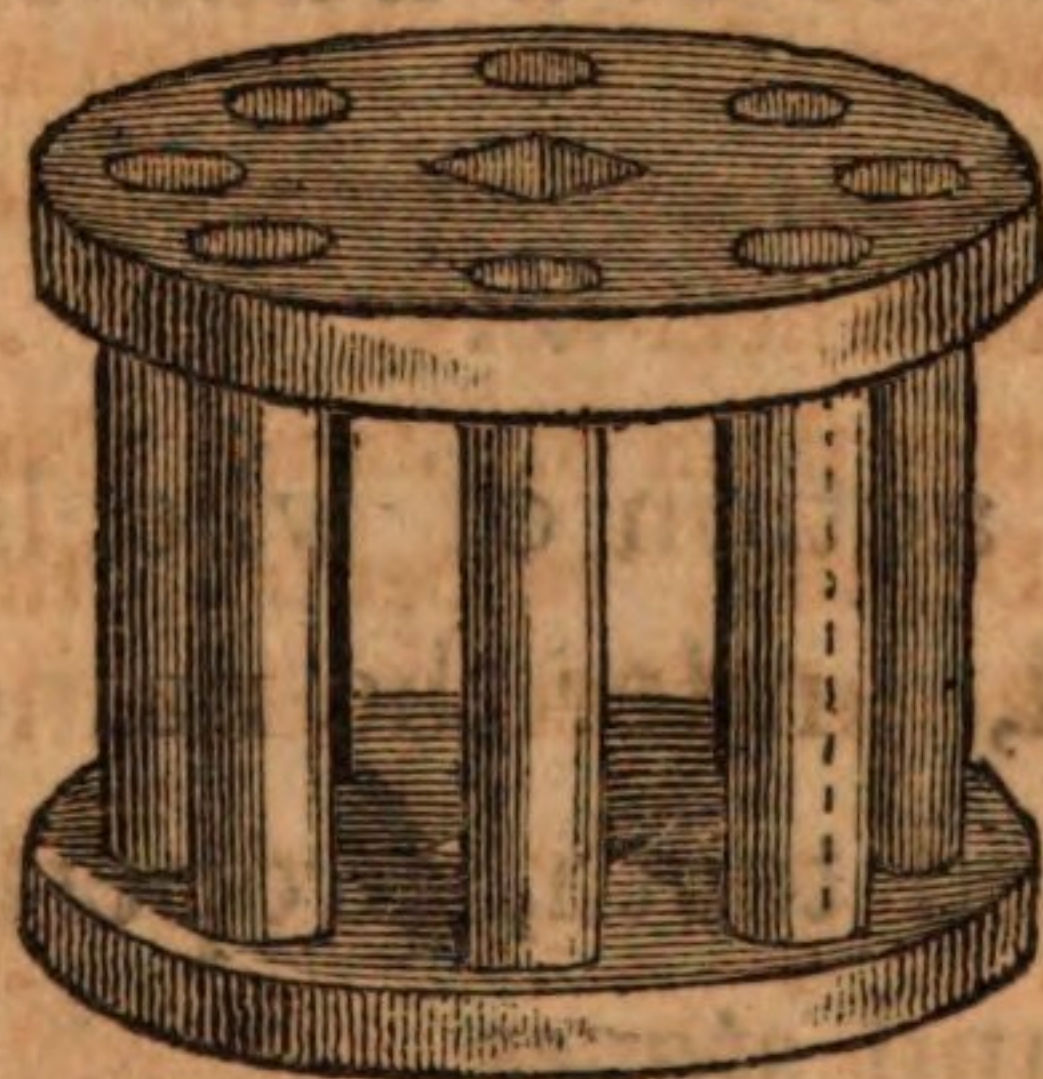
GENERAL DEFINITIONS.

I.

WHEN two toothed wheels act upon one another, the greater is called the *Wheel*, and the lesser the *Pinion*.

II.

Instead of the pinion, the trundle is sometimes used, such as is here represented. It is likewise known by the names of lantern and wallower.



III.

As pinions and trundles are employed for the same purposes, when the action of two wheels is spoken of, in general, the trundle is comprehended under the name pinion.

IV.

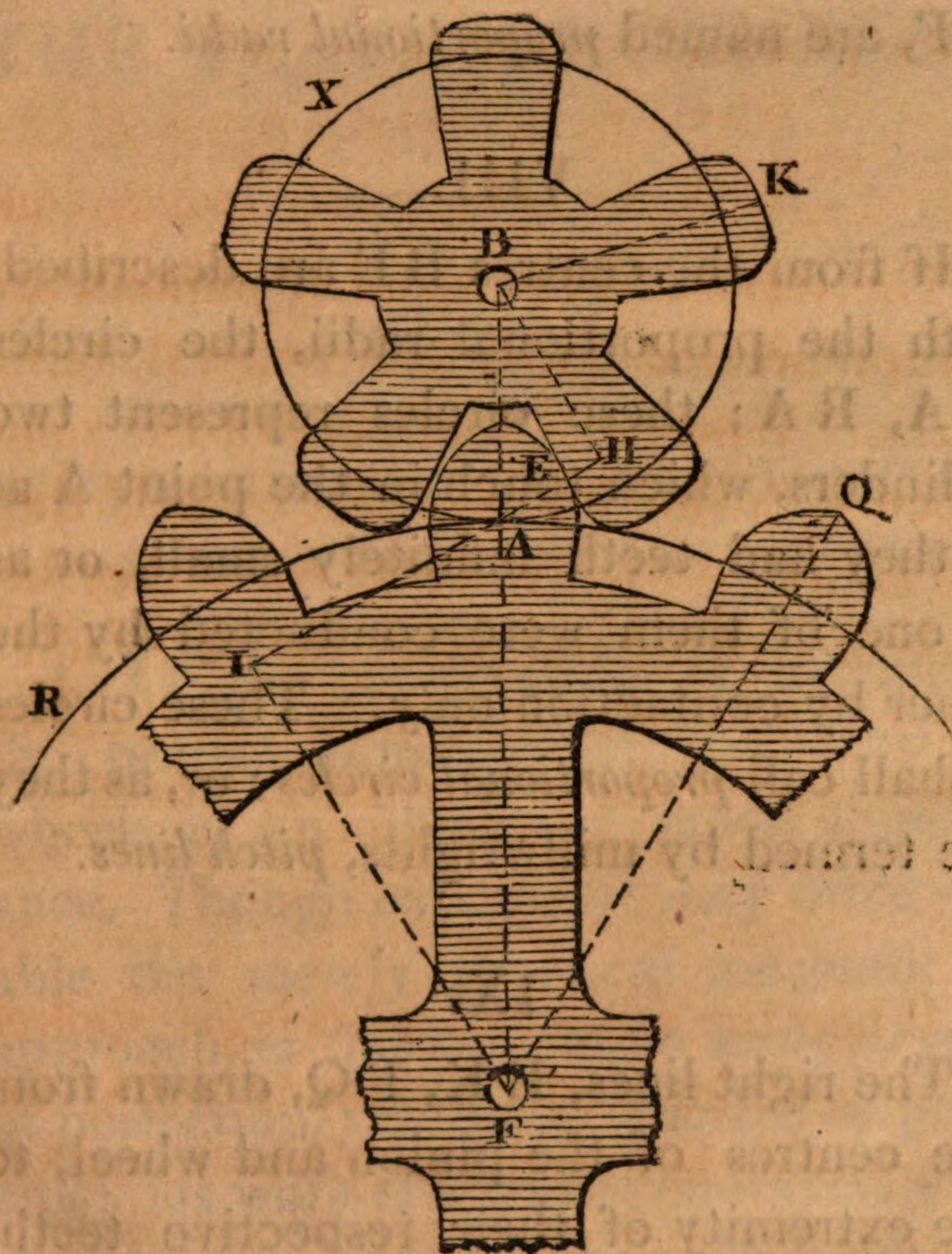
The teeth of wheels and of pinions, are comprehended under the general term, *Teeth*. Where the teeth are of the same piece with the body of the wheel, they are called, properly, *teeth*; when they are each of a particular piece, they are called *cogs*. The teeth of pinions are called *leaves*, and those of a trundle, *staves*.

V.

When the action of wheels is spoken of in general, under the name *teeth*, are comprehended teeth, properly so called, *cogs*, *leaves*, and *staves*.

VI.

The straight line B F, which joins the centres B F of a pinion and wheel, which act together, is called the *line of centres*.



VII.

When the line of centres BF is divided into two parts, AB , AF , proportional to the number of the teeth in the wheel, and in the pinion, these two parts, AB , AF , are named *proportional radii*.

VIII.

If from the centres B F are described, with the proportional radii, the circles XA , RA ; these circles represent two cylinders, which touch in the point A as if they had teeth infinitely small, or as if one of them were conducted by the other by contact only. These circles I shall call *proportional circles*; or, as they are termed by millwrights, *pitch lines*.

IX.

The right lines, BK , FQ , drawn from the centres of the pinion and wheel, to the extremity of their respective teeth, are called, *real radii*.

AN ESSAY
ON
THE TEETH OF WHEELS.

CHAP. I.

OF THE PRINCIPLES OF THE CONFIGURATION OF THE TEETH OF WHEELS.

1. IN the construction of machines, the proper formation of the teeth of wheels is an object of much importance. Though experience may often enable the merely practical mechanic to approach, in this respect, to some degree of perfection, yet, being ignorant of principle, his work is always conducted with uncertainty, and he generally produces

machines, expensive in working, and defective in regularity, effect, and duration.

For when the acting parts of a machine are not truly formed, it may be so loaded, as just to be in equilibrio with its work in the most favourable situation of its parts, but when it changes into a less favourable situation, the machine will stop, or, at least, stagger, hobble, or work unequally.

The best figure, therefore, which can be given to the teeth, is that which shall cause them always to act equally and similarly, in situations equally favourable, and which shall consequently give the machine the property of being moved uniformly by a power constant and equal; or, in other words, ensure an uniformity of pressure and velocity.

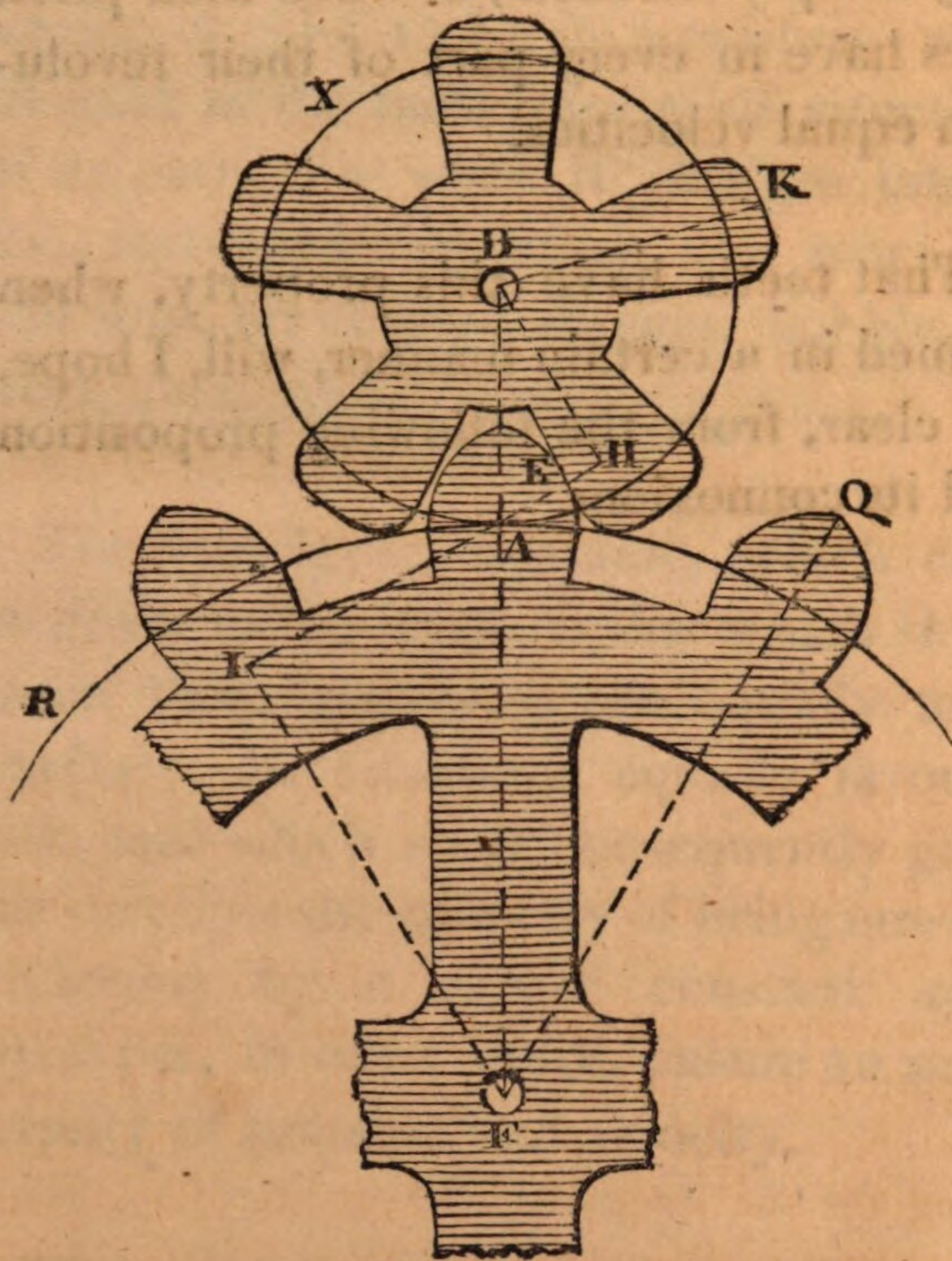
Were the teeth of wheels infinitely small, their action would be regarded as that of cylinders, simply touching, having

the property required. The finite and sensible teeth given to wheels will, therefore, be of the most advantageous figure, when one wheel conducts another, as if they simply touched ; or when their pitch lines have in every part of their revolution equal velocities.

That teeth have this property, when formed in a certain manner, will, I hope, be clear, from the following proposition and its connexions.

PROPOSITION.

2. When teeth are of such a form, that a perpendicular HEI^* drawn to



* For the manner of drawing this perpendicular, see Corollary III. p. 26.

the tangent to the edge of the tooth in the point of contact E, cuts the line of centres at the termination A, of their proportionate radii, *their pitch lines shall have in corresponding places, equal velocities*, whether the wheel drives the pinion, or the pinion the wheel; that is to say, that they will move each other as if they merely touched*.

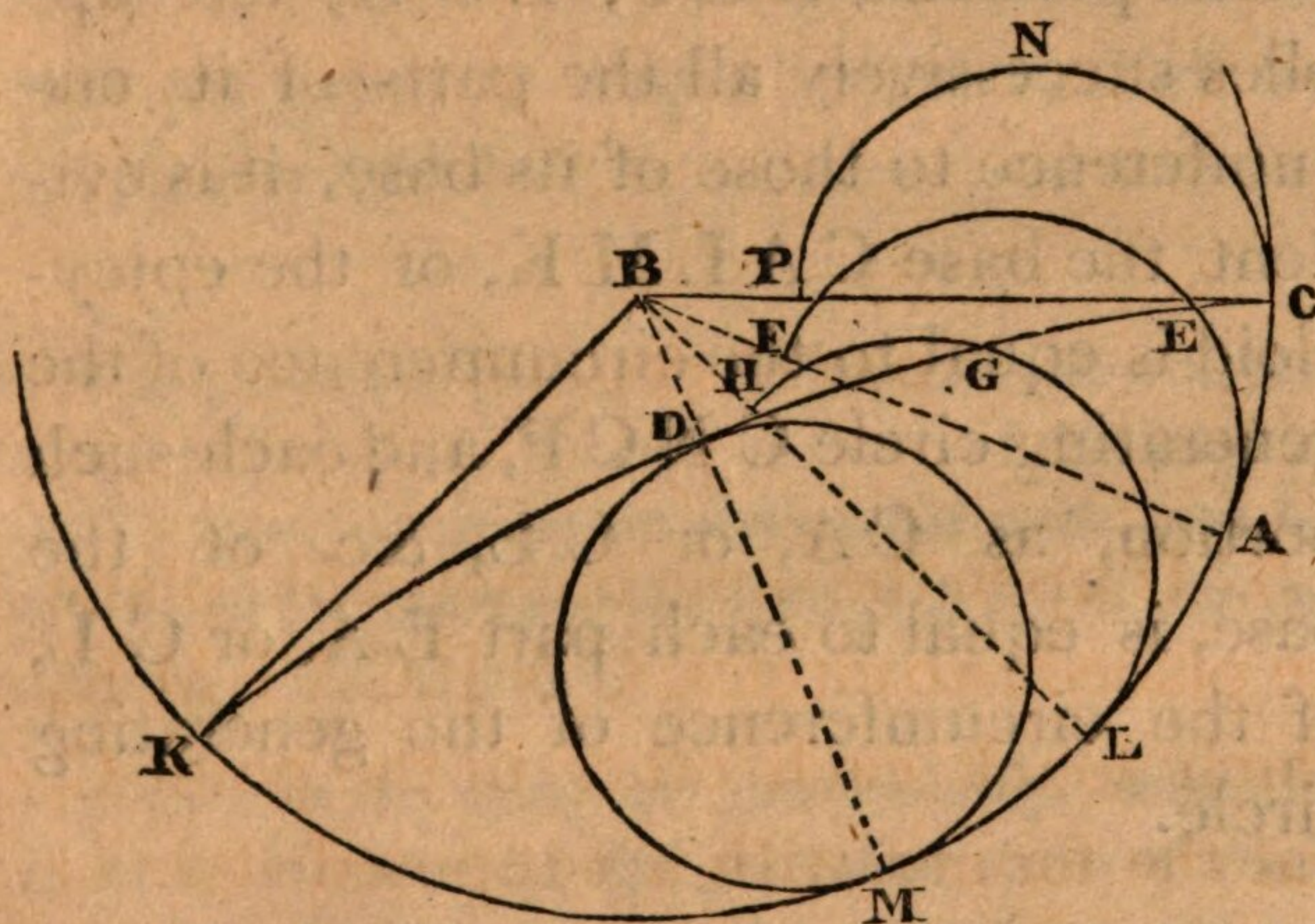
I shall now proceed to show, that the epicycloid gives the property to the teeth of wheels required in the preceding proposition, and shall begin with some definitions respecting that curve.

* This being a fundamental proposition, it is of importance that it should be well understood; I shall therefore in this note attempt a popular illustration of it.

It is demonstrable that the line, H B, has the same proportion to the line I F which A B has to A F. Now let us suppose H B and I F to be levers, and H I a string, the one lever pressing from the other, would act upon it with just the same force that the pinion and wheel do at the point A, where the pitch lines touch; or, in other words, as if the circle X acted on the circle R, by means of a string, as pulleys do on each other by a band.

II.

The circle C N P, which, in revolving, describes the epicycloid, is called the *generating circle of the epicycloid*, and the arc C A L M K of the immoveable circle, upon which the generating circle revolves, is called the *base of the epicycloid*.



III.

When the generating circle revolves without the circle of its base, as in p. 20, the epicycloid is called an *exterior epicy-*

cloid; and when the generating circle rolls within the circle of its base, as in p. 21, the epicycloid is called an *interior epicycloid*.

COROLLARIES.

I.

4. As the generating circle in revolving from its first situation, C N P, to different portions, A E F, L G H, &c. applies successively all the parts of its circumference to those of its base, it is evident the base C A L M K, of the epicycloid is equal to the circumference of the generating circle C N C P, and each such portion, as C A, or C L, &c. of the base, is equal to each part E A, or G L, of the circumference of the generating circle.

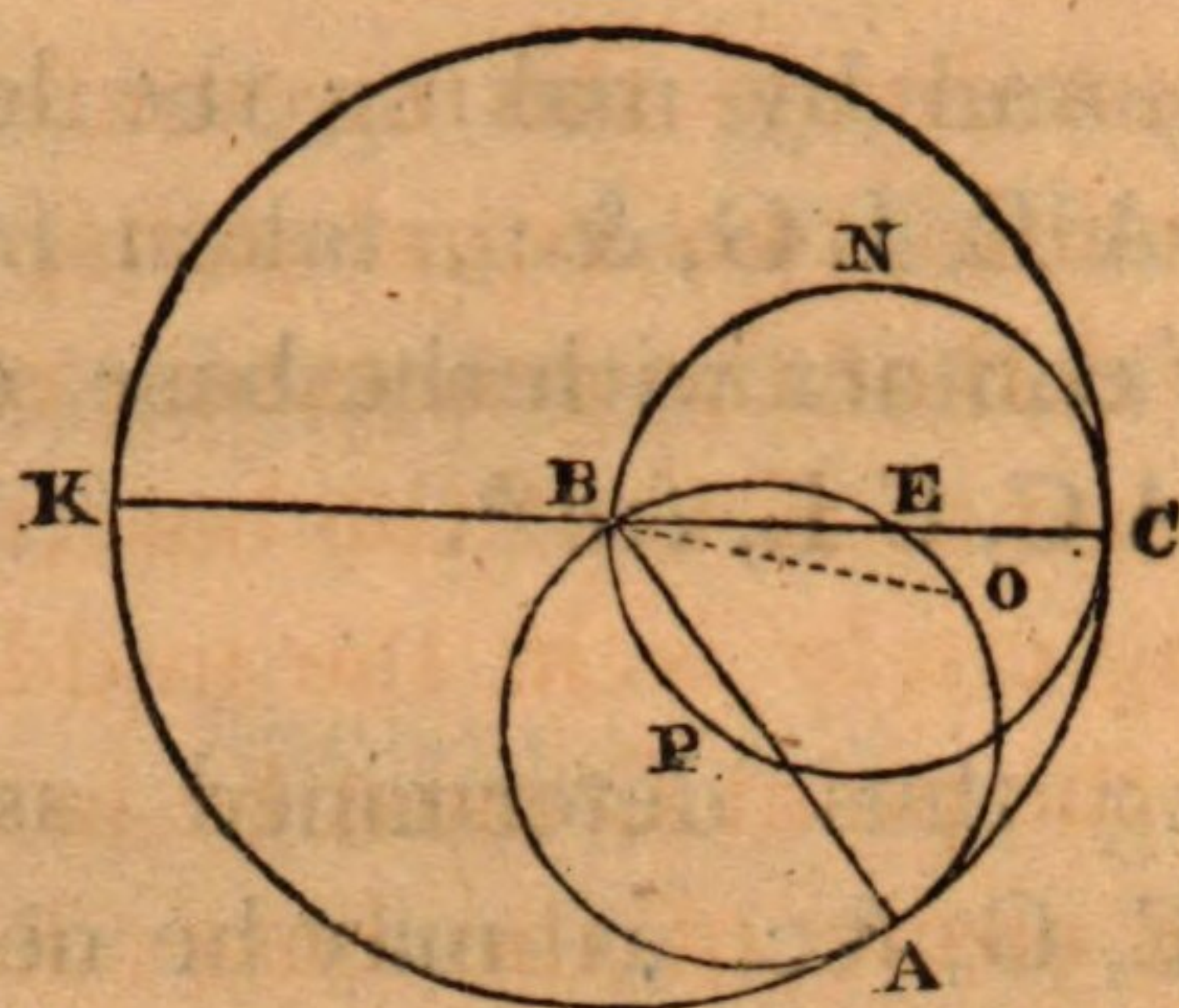
Hence a method of drawing the epicycloid, by describing the circles A E F, L G H, &c. which have all the same radii as the generating circle C N P, and touch the base C A L M K in any points

A, L, &c ; and by making the length of the arcs A E, L G, &c., taken from the points of contact with the base, equal to the arcs A C, C L, &c*.

Having thus determined as many points, E, G, &c. as may be necessary, the curve C E G D K, which shall pass through them and the point C, where the supposed style of the generating circle was supposed to begin its tract, shall be an epicycloid †.

* In practice, this is most easily done, and with sufficient accuracy, by dividing each arc of the base, as at A C, into a number of small equal parts, and by setting off the same number upon each arc of the generating circle, as at A E.

† To draw the epicycloid mechanically, make the circle of the base and the generating circle of wood, and having fixed a tracer in the circumference of the generating circle, let the base remain at rest, and the tracer, during the rolling of the generating circle, will draw an epicycloid.—In order to make circles move with more accuracy, a small piece of tape may have one of its ends nailed to the circumference of the one circle, and the other end to the other circle.



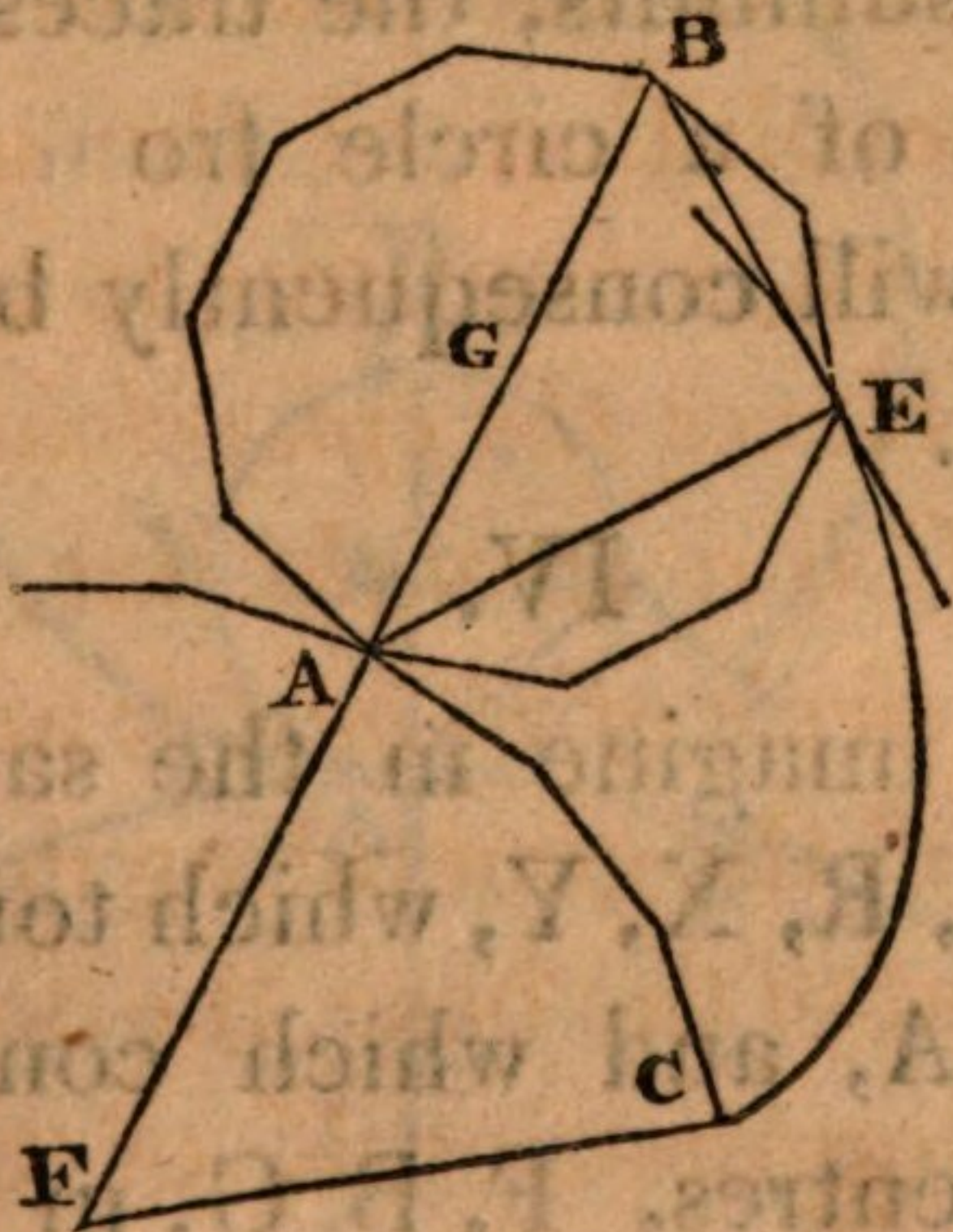
II.

5. When the generating circle CNP revolves within the circle of its base, and has for its diameter the radius BC of its base, the point C , the place of the style during the revolution of the generating circle, will always continue in the diameter CBK . Hence the epicycloid described by the style C is a straight line, and a diameter of the circle of its base*; and the circumference of the generating circle CNP being half that of the base, the commencement C and termination K of the epicy-

* Upon this principle a parallel motion has been constructed. It is used by Messrs. Fenton, Murray, and Wood, in some of their smaller steam engines.

For a short account of it, see Gregory's Mechanics, vol. ii. p. 265.

cloid, must divide the circumference of the base into two equal parts, and the diameter AB of the generating circle being half that of $K C$ of the base, when the generating circle is in the middle of its progress, the point C must be in the centre of the circle of the base; hence we have a point at the origin, in the middle, and at the end, of the epicycloid, which all lie in the diameter of the base, and the whole epicycloid may be considered as coinciding with the diameter of the base*.



* It would carry me too far into mathematics for many readers, were I strictly to demonstrate, that every point of the epicycloid must lie in the diameter of the base; what is said however will, I hope, satisfy them of the truth. The mathematical reader will find

III.

6. When the generating circle of the epicycloid, as in p. 25, is in any position, A, E, B, touching the circumference of its base in any point A, *a straight line, drawn from the point of contact A to the point E, actually describing the epicycloid, will be perpendicular to it.*

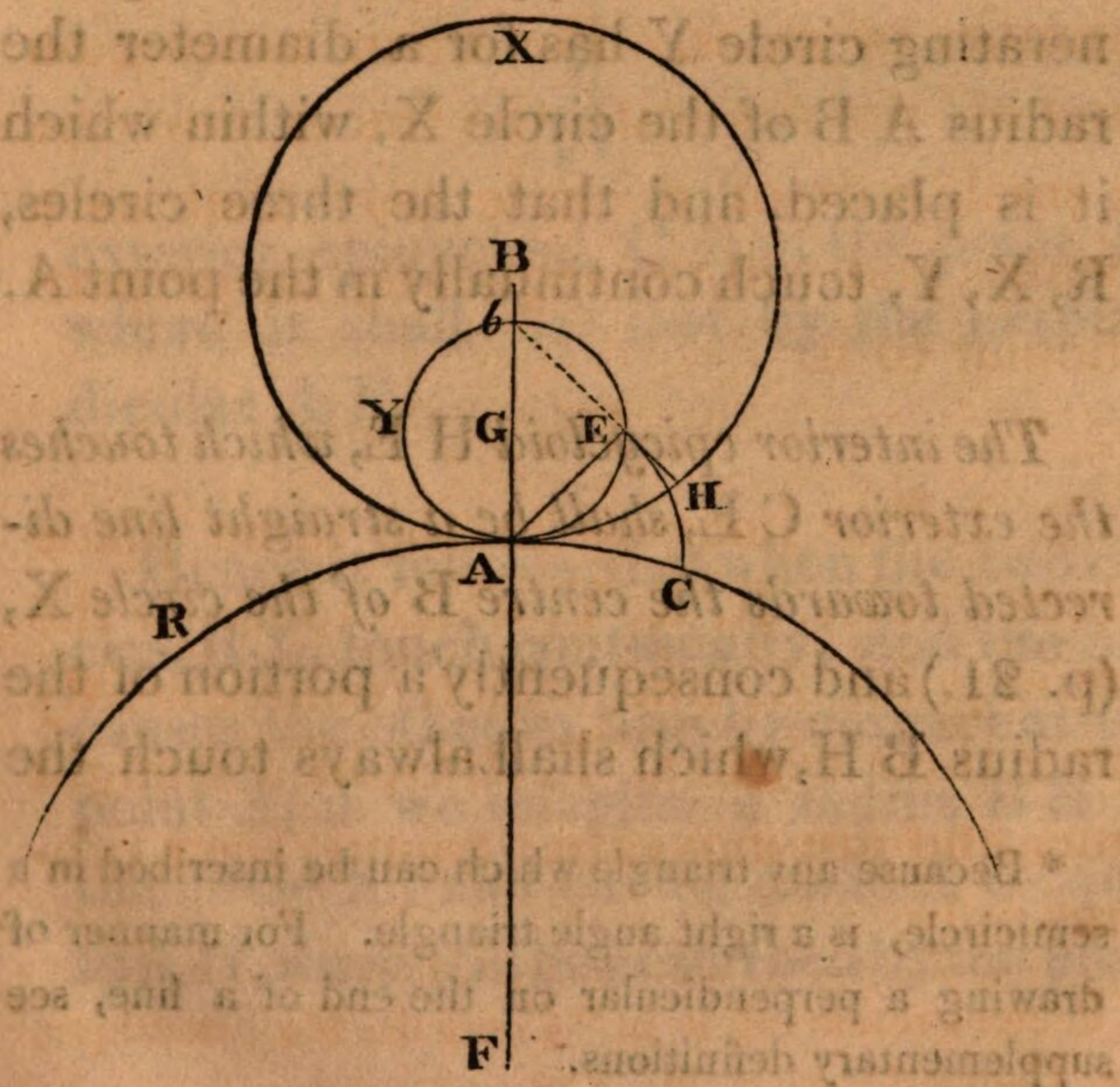
This will be evident by supposing the circles to be polygons, having a great number of sides. For when turning on any of the summits, the traces describe a small part of a circle from that as a centre, and will consequently be perpendicular to it.

IV.

7. Let us imagine in the same plane three circles, R, X, Y, which touch in the same point A, and which consequently have their centres, F, B, G, in a straight line, and are moveable round their centres only.

a demonstration of it in "Cours de Mathematique, par Camus," tom. iv. No. 538.

Suppose a style fixed in the circumference of the circle Y, and that the three circles are made to turn by the movement of one of them: if we make each of the arcs, A H, A C, equal to A E, then the style placed in E shall have described on the plane of the circle R, a portion C E of an exterior epicycloid, and on the plane of the circle X, a portion H E of an interior epicycloid.



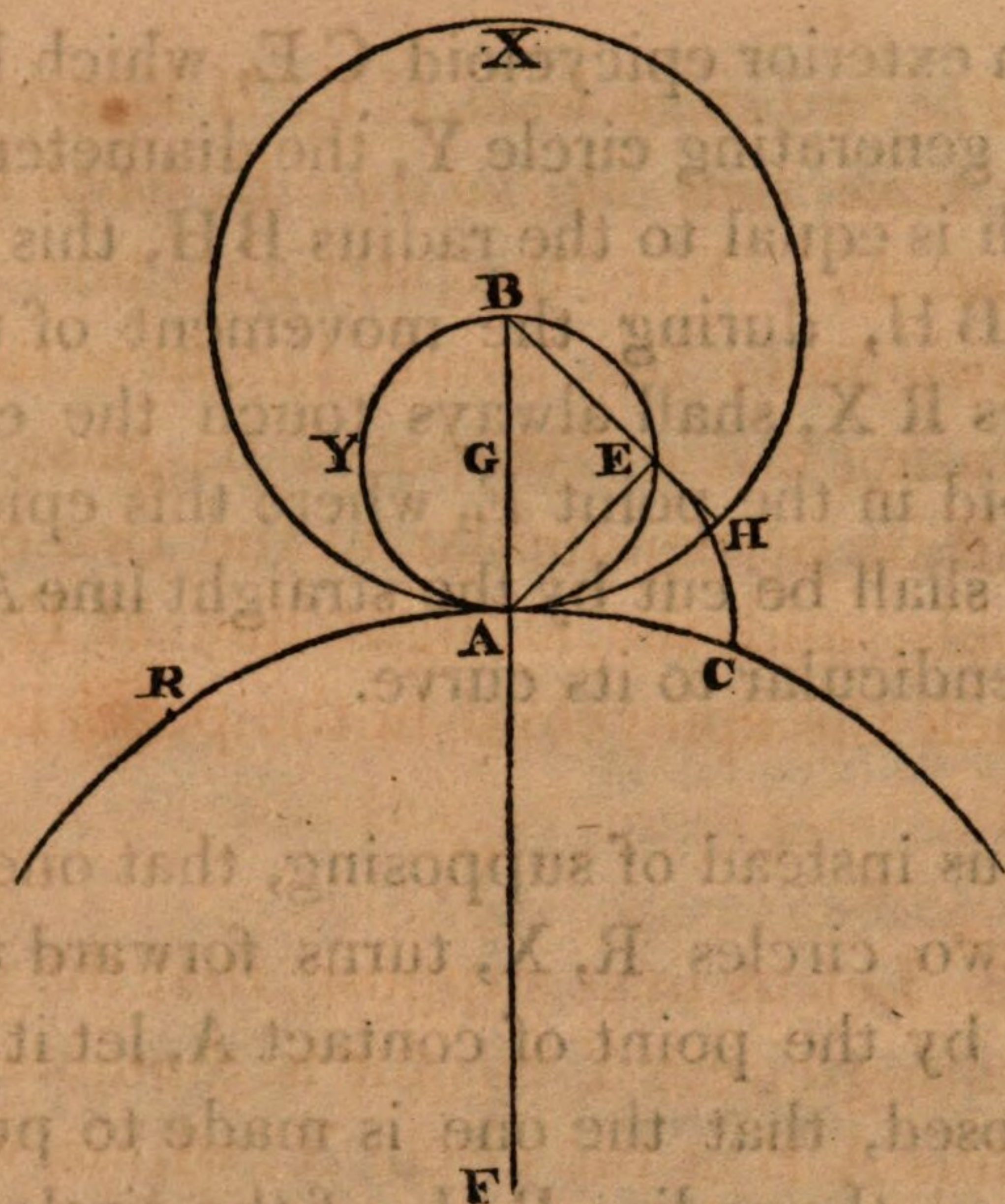
The two epicycloids, C E, H E, traced in the same time by the style E, touch in the point E. For the straight line A E, drawn from the point A, where the generating circle Y touches its base R C, shall be perpendicular to the two epicycloids, and the straight line H E shall touch the epicycloid in the point E.*

V.

8. Let us next suppose, that the generating circle Y has for a diameter the radius A B of the circle X, within which it is placed, and that the three circles, R, X, Y, touch continually in the point A.

The interior epicycloid H E, which touches the exterior C E, shall be a straight line directed towards the centre B of the circle X, (p. 21.) and consequently a portion of the radius B H, which shall always touch the

* Because any triangle which can be inscribed in a semicircle, is a right angle triangle. For manner of drawing a perpendicular on the end of a line, see supplementary definitions.



exterior epicycloid C E in the point E, where it shall be met by the perpendicular A E.

Hence it follows, that when the two circles, R X, touch continually, and the one causes the other to turn by contact at the point A, if we imagine a radius B H in the circle X; and having made A C equal to A H, there will be described by the point

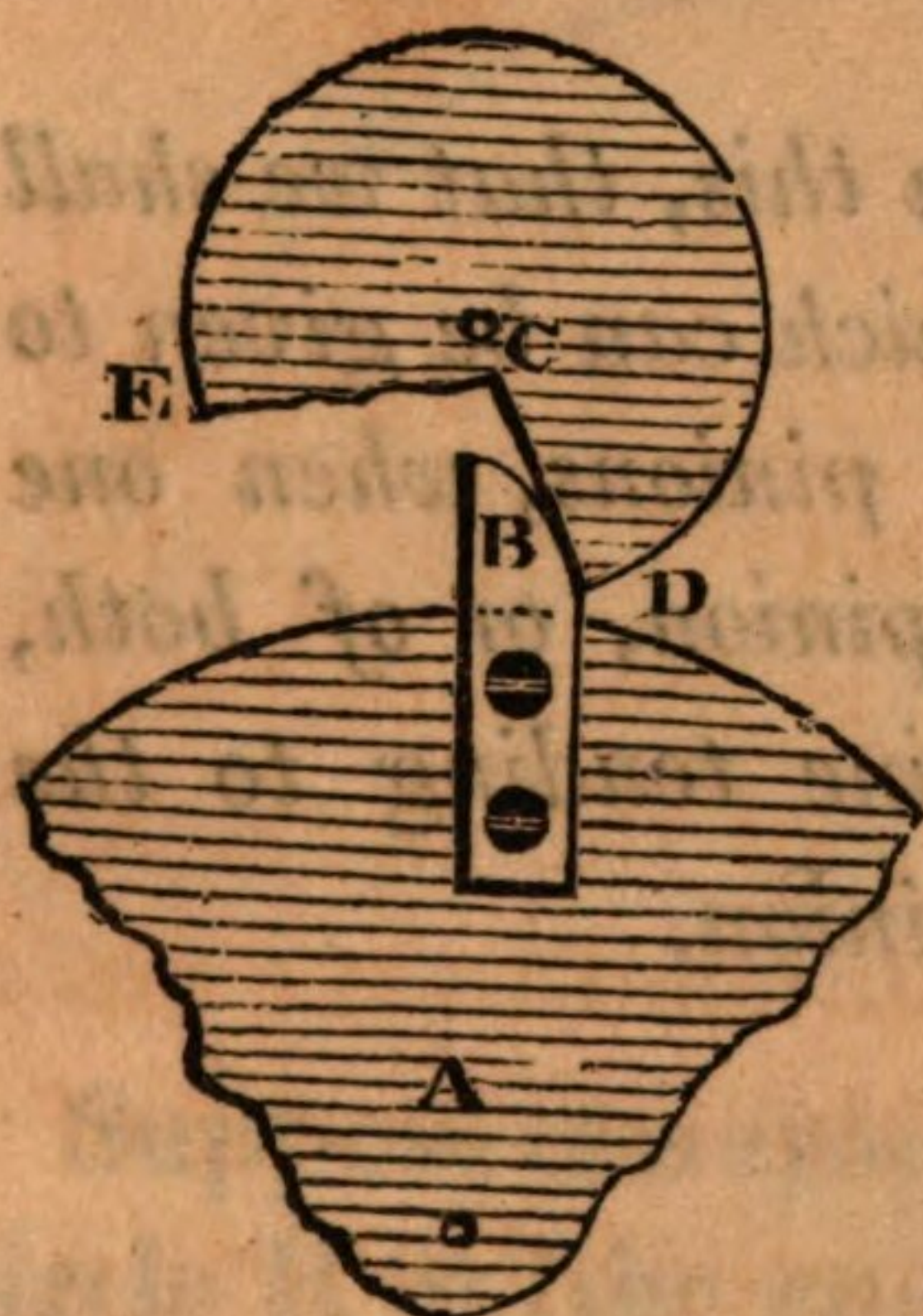
C, an exterior epicycloid C E, which has for a generating circle Y, the diameter of which is equal to the radius B H, this radius B H, during the movement of the circles R X, shall always touch the epicycloid in the point E, where this epicycloid shall be cut by the straight line A E perpendicular to its curve.

Thus instead of supposing, that one of the two circles R, X, turns forward the other by the point of contact A, let it be supposed, that the one is made to push forward the radius B H, of the circle X, by an epicycloid C E attached to the circle R, and described by the movement of the circle Y, the diameter of which is equal to the radius B H.

One may be able thus reciprocally to make the epicycloid C E, attached to the circle R, push forward by a radius B H a circle X ; and by means of the epicycloid C E, and of the radius, B H, the

two circles, R, X, may be able to conduct themselves as if put forward by the point of contact A*.

For suppose the radius, B H, and the epicycloid, C E, to be teeth of wheels,



* To be satisfied of this experimentally, make any two circles of wood; to the circumference of one of them A, fix a piece of wood B, formed into an epicycloid, generated by a circle half the diameter of C upon A as a base.

On the circle C, draw the line C D, and cut out the part bounded by that line and C E.

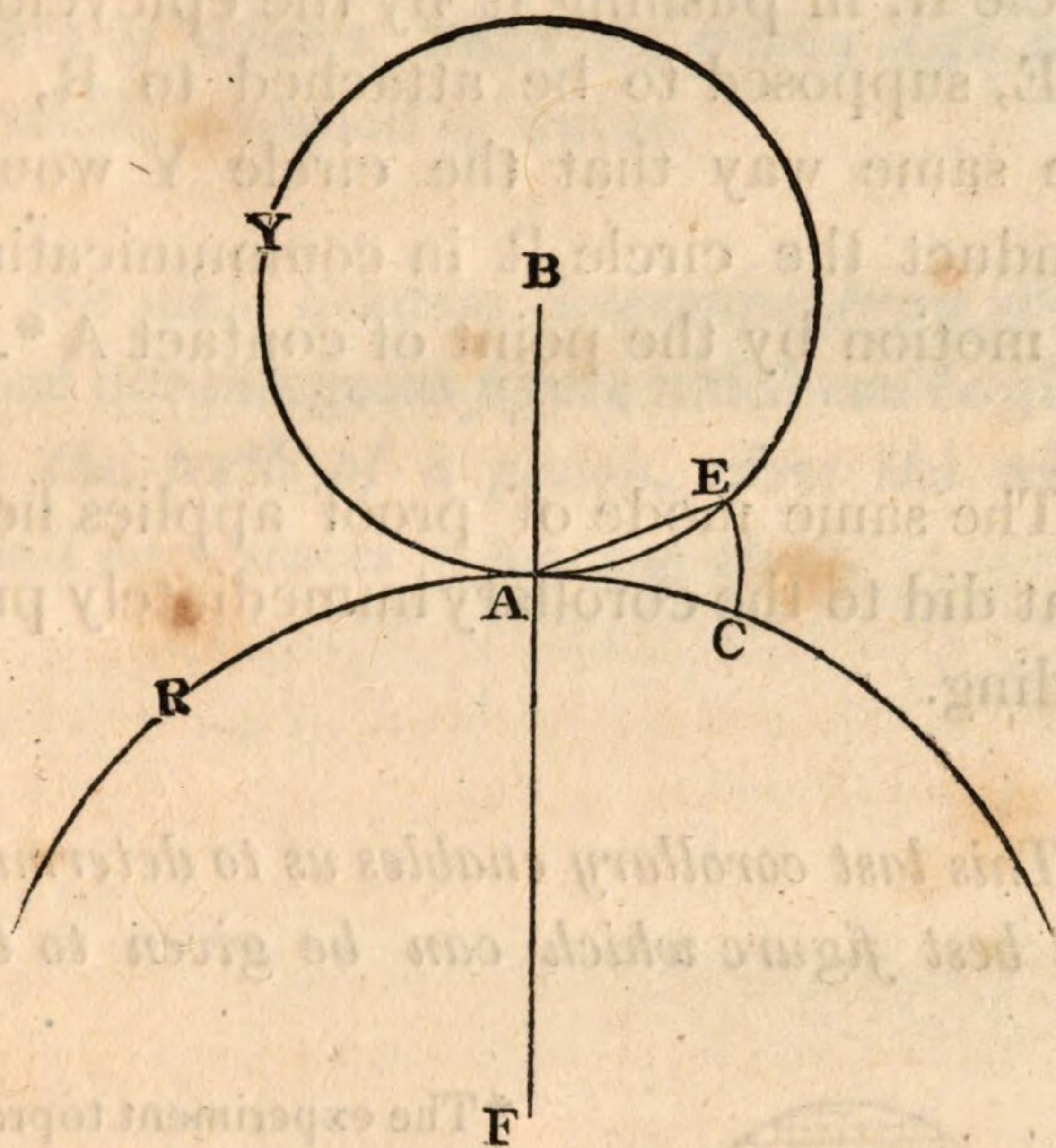
If you cause one of the circles to move the other by the parts B, C, D, both circles will have the same velocity; as may be ascertained by putting a mark opposite any point in the circumference of each circle before they begin to move, and another after they stop, and the distance between which, measuring by the arcs, will be found equal.

X and Y ; the perpendicular A E, from the touching surfaces in all situations, cuts the line of centres at the termination A of their proportional radii. But we saw p. 19, that when this was the case, the proportional circles must have equal velocities.

It is principally from this, that we shall deduce the best figure which can be given to the teeth of wheels and pinions, when one part of the wheel and pinion, or of both, ought to be a straight line tending to the centre of such wheel or pinion.

VI.

9. If in the same plane we have but two circles, R, Y, which touch in the point A, and if the movement of the one, communicate itself to the other, by this point of contact, any point E of the circumference of the circle Y, describes upon the plane of the moveable circle R, an epicycloid C E.



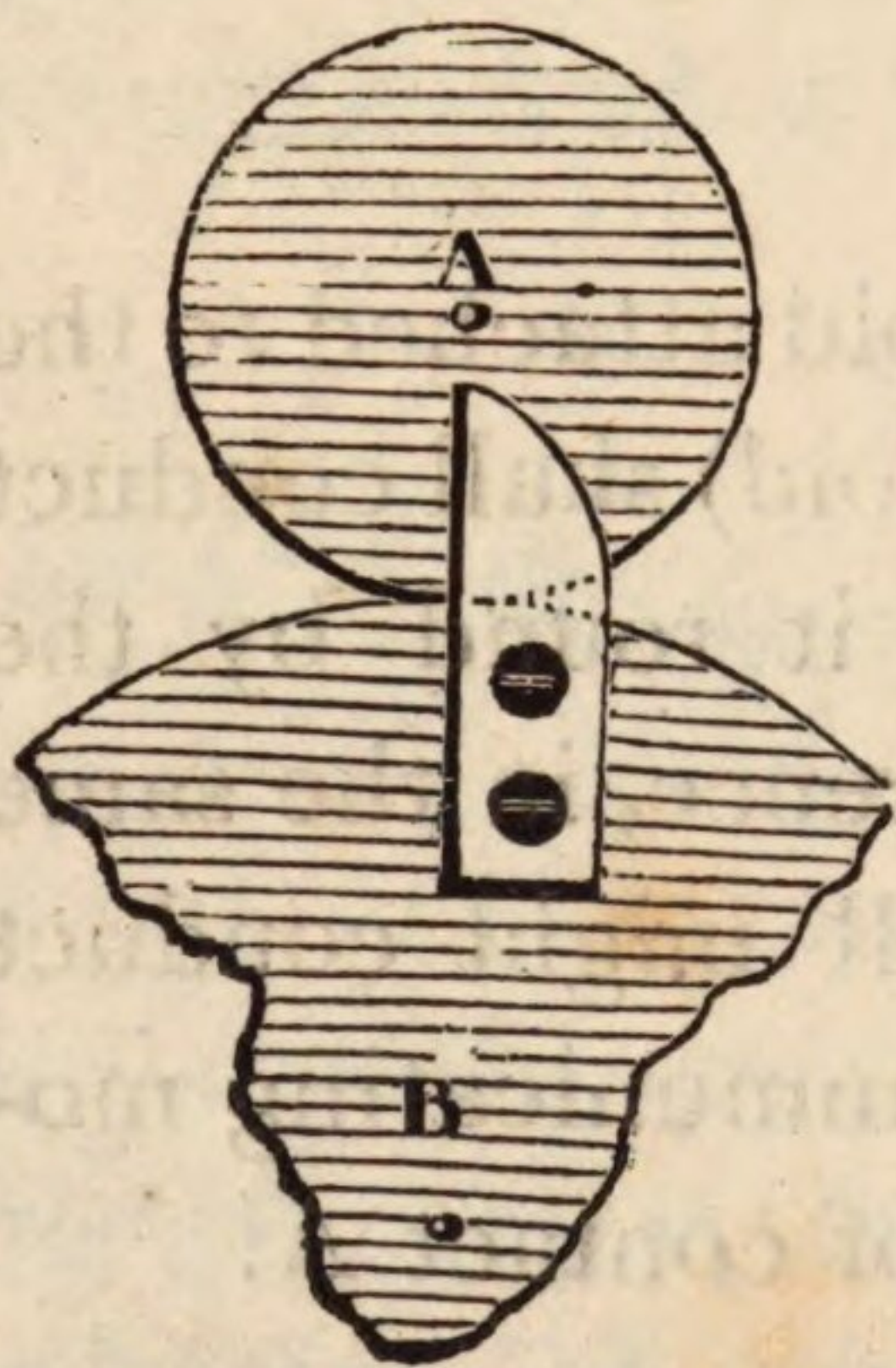
Suppose this epicycloid attached to the circle R, it (the epicycloid) shall conduct the circle Y, pushing it round by the point E of its circumference, in the same manner as the circle R might conduct the same circle Y in communicating motion to it by the point of contact A.

And in like manner, the point E of the circumference of the circle Y, turns the

circle R, in pushing it by the epicycloid C E, supposed to be attached to R, in the same way that the circle Y would conduct the circle R in communicating its motion by the point of contact A *.

The same mode of proof applies here that did to the corollary immediately preceding.

This last corollary enables us to determine the best figure which can be given to the



* The experiment to prove this is similar to the former, but with this difference, that in the circumference of one of them, A, is fixed a fine needle, which is made to act against a piece of wood, formed into an epicycloid, fixed upon the other, B, which epicycloid is generated by A upon B as a base.

teeth of wheels, when the pinion shall be a trundle composed of staves.

We shall likewise determine from it the most advantageous figure which can be given to the teeth of a pinion, when the wheel shall have staves in place of teeth.

CHAP. II.

OF THE APPLICATION OF THE PRINCIPLES OF THE CONFIGURATION OF THE TEETH OF WHEELS.

1. Having endeavoured to show, that an epicycloid is a curve, whereby two circles may conduct themselves as if put forward by the simple contact of their circumferences, I shall now attempt a practical explanation of this curve, in giving the best form to the teeth of wheels.

SECTION I.

Of Spur Geers.

2. By *Spur Geers* is understood wheels acting together, with their axes parallel and in the same plane; under this head

the wheel and trundle come first to be considered.

Of the Wheel and Trundle.

3. To determine the figure of the teeth of the wheel, which depends always upon that of the staves of the trundle, we shall first suppose the staves indefinitely small, and represented (p. 39.) on the end of the trundle by the points A, E, H, &c.: when we have found the figure of the teeth proper to conduct the indefinitely small staves, (which are used for demonstration only), we shall, by means of that figure, trace the true form which should be given to the teeth of wheels to conduct trundles with cylindric staves of some magnitude. Thus the solution of this case, naturally divides itself into two parts.

I.

To find the figure of the teeth when the staves are indefinitely small.

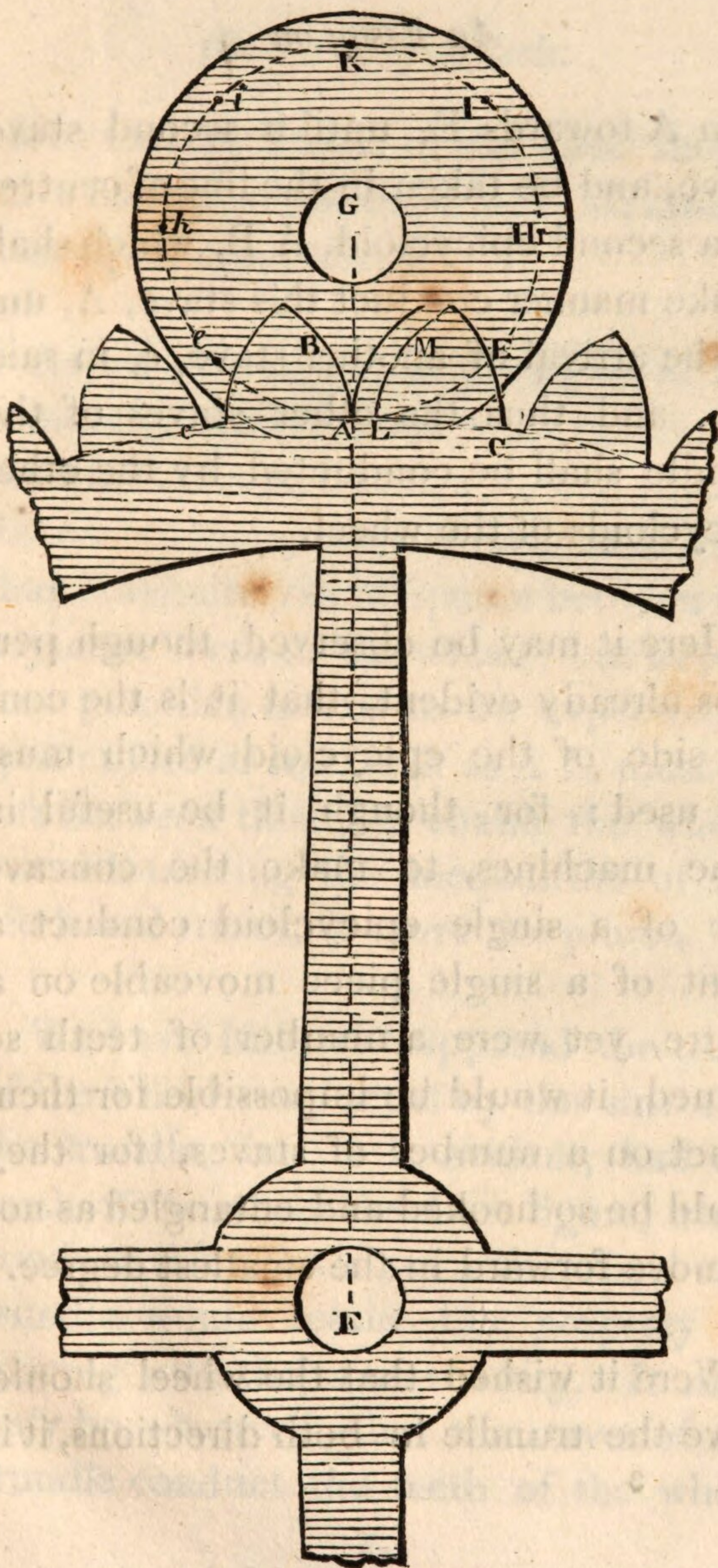
Draw the proportional circles, $c A C$ and $e A E$, and divide each of them into the number of equal parts which it should have of teeth *.

We have seen †, if the circle, $c A C$ which touches the circle, $e A E$ have attached to its circumference an epicycloid, C, E , described by the point E of the circumference of the circle, $e A E$ rolling upon the circle $c A C$, the epicycloid conducts the circle, $e A E$ by the point E , as if conducted by contact at A , and consequently the circumferences of the two circles shall have the same velocity.

The epicycloid, C, E , is then the best figure which can be given to the teeth of a wheel to conduct a trundle, the staves of which are indefinitely small, and therefore must move the stave E , in the direction

* This operation is called by millwrights *setting off the pitch*.

† See Chap. i. Article 9.



from A towards E, until a second stave arrive, and be taken in the line of centres by a second epicycloid, A B, which shall in like manner conduct this stave, A, until the arrival of another stave, e, in said line: and thus the other staves of the trundle shall be conducted by the other epicycloids of the wheel.

Here it may be observed, though perhaps already evident, that it is the convex side of the epicycloid which must be used: for though it be useful in some machines, to make the concave side of a single epicycloid conduct a point of a single piece moveable on a centre, yet were a number of teeth so formed, it would be impossible for them to act on a number of staves, for they would be so hooked and entangled as not to move forward in the smallest degree.

Were it wished that the wheel should move the trundle in both directions, it is

vious that each tooth of the wheel should have its opposite sides, C E, L M, formed into equal epicycloids.

As we have supposed the staves of the trundle indefinitely small; were the teeth of the wheel also perfect figures, and equally distanced, there would be no need of other than indefinitely small spaces between the adjacent teeth of the wheel; but as perfect precision is not to be expected, a space more or less, such as A L, must be left between them, to enable the wheel, notwithstanding the inequalities of the teeth and staves, to move the pinion.

We have hitherto supposed the teeth of the wheel conducted by the staves of the trundle, but it is evident, had the teeth of the wheel the same figure, when conducted by the staves, the wheel and trundle would retain the property of moving with the same velocity. It may only be observed, that the staves of the trundle conduct the teeth of the wheel

in approaching the line of centres, while the teeth of the wheel conduct the staves of the trundle in their progress from that line *.

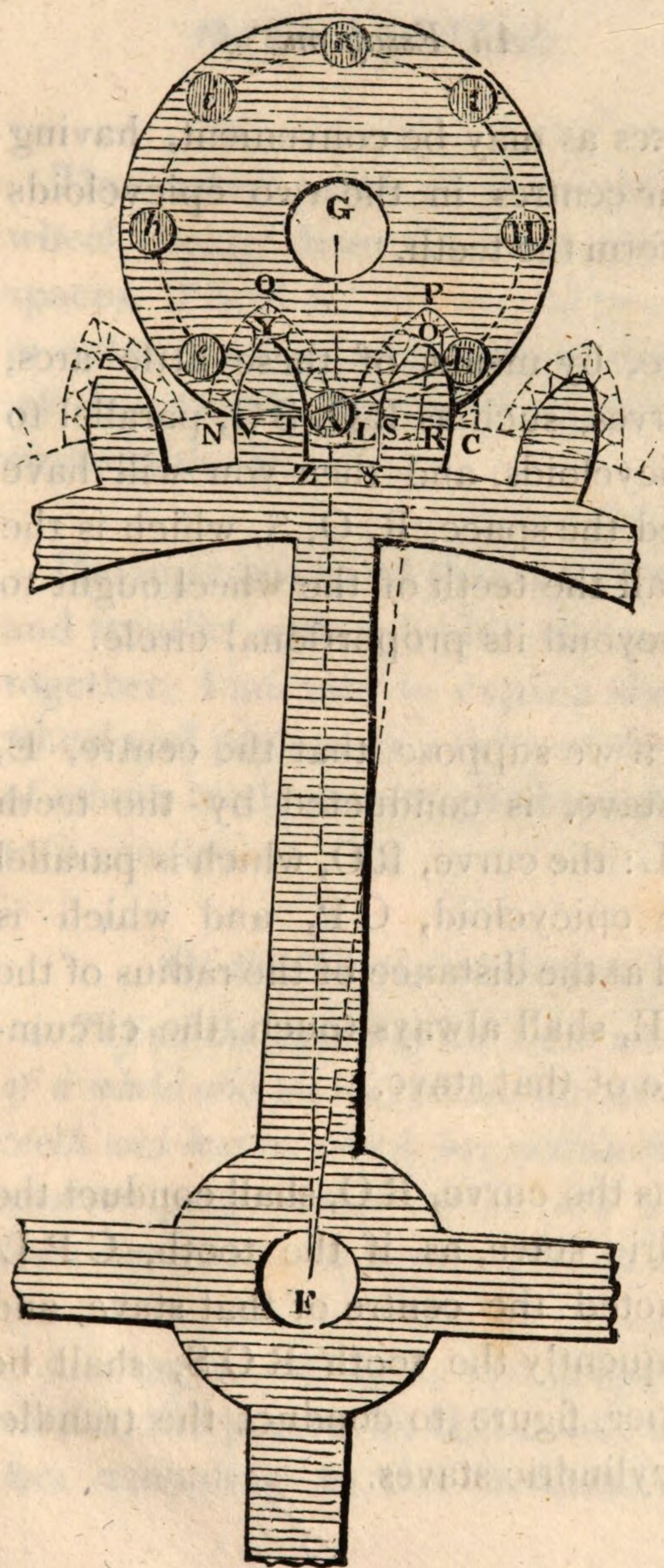
II.

To find the figures of the teeth of the wheel, when the staves of the trundle are cylinders of a finite diameter.

Consider the trundle at first as having infinitely small staves, represented by the centres of the staves, A, E, H, &c. and trace, as above mentioned, the teeth C L P, A Q N, &c. of the wheel, as if it had to conduct a trundle with infinitely small staves: observing to leave a small space, such as A L, between all the teeth, in order that they may act freely.

Describe, with the radius of the staves, upon the plane of each tooth, as many

See Article 6 of this chapter.



small arcs as may be convenient, having all their centres in the two epicycloids which form the teeth,

Trace, by means of these little arcs, two curves, such as RO , SO , parallel to the epicycloids, and then you will have inclosed the space, R , O , S , which is the figure all the teeth of the wheel ought to have beyond its proportional circle.

For if we suppose, that the centre, E , of a stave, is conducted by the teeth C , P , L : the curve, RO , which is parallel to the epicycloid, CP , and which is placed at the distance of the radius of the stave E , shall always touch the circumference of that stave.

Thus the curve, RO , shall conduct the cylindric stave, as if the tooth, $CP L$, conducted the centre of that stave, and consequently the tooth ROS , shall be a proper figure to conduct the trundle, with cylindric staves.

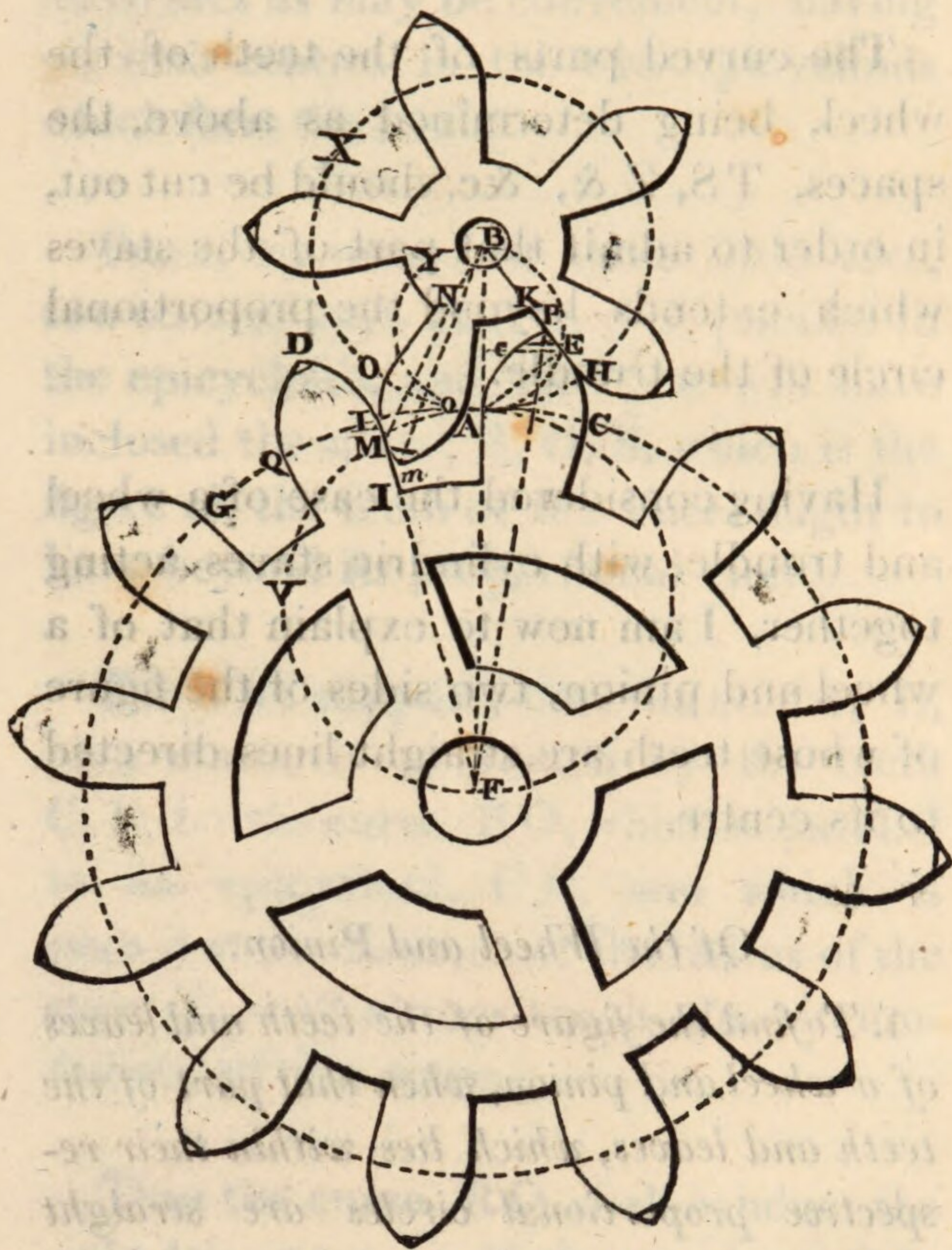
The curved parts of the teeth of the wheel, being determined as above, the spaces, TS, Z &, &c. should be cut out, in order to admit that part of the staves which extends beyond the proportional circle of the trundle.

Having considered the case of a wheel and trundle, with cylindric staves acting together, I am now to explain that of a wheel and pinion, two sides of the figure of whose teeth are straight lines directed to its centre.

Of the Wheel and Pinion.

4. *To find the figure of the teeth and leaves of a wheel and pinion, when that part of the teeth and leaves, which lies within their respective proportional circles are straight lines directed to the centres of these circles.*

Having set off upon the proportional circles, the points, G, Q, L, and, O, o, H, &c. according to the thickness of the



teeth and leaves, draw lines from these points, tending towards the centre of their respective circles, to serve as the sides of the spaces between the teeth, and between

the leaves, the depth of which spaces must be such as to give room for the action of the curved parts of the teeth and leaves.

Then describe upon the extremities of the sides of each tooth, epicycloids, such as Q D, L D, with the generating circle Y, the diameter of which is equal to the proportional radius of the pinion, upon the circumference of the proportional circle of the wheel as a base.

The mode of forming the teeth being thus shown, that of the leaves will be plain, V being the generating circle of their epicycloid, upon the circumference of the circle of the proportional pinion as a base.

We have seen *, if the radius, B H, of the proportional pinion, be pushed by an epicycloid C P, generated by the circle Y, upon the pitch line of the wheel, and

* Chap. I. Article 8.

projecting therefrom, the pinion shall turn with the same velocity as the wheel.

In the same manner it may be proved, that the same effect will be produced, if the epicycloid, O, M, m , attached to the pinion, be pushed towards the line of centres, by the radius, LF , of the wheel.

Lastly, the two opposite sides of the teeth, and those of the leaves, ought to have the same figure, for the ease of action, and to give the wheel and pinion the liberty of being moved in either direction.

From these principles, I hope, it will be evident, that the figure here given to the teeth, will make the wheel and pinion move with perfect regularity.

REMARKS.

5. As it is the curved part of the teeth of the wheel, that should push the straight

flank, H K, of those of the pinion, in removing from the line of centres, and as the point, E, where the flank is acted upon, is a perpendicular drawn from A, it shall be always that by which the wheel shall push, it is clear, that when the extremity, P, of the epicycloid, C P, reaches the point E, it shall cease to move the tooth H K; if the extremity, P, arrive at the point E, before the flank, O N, of the following tooth of the pinion has reached the line of centres, the curved part, O, M, m, of this tooth, must be pushed by the straight flank, L I, of the following tooth of the wheel, till the flank, O N, reaches that line: so that in this case, the wheel conducts the pinion, at one time before, and, at another, beyond the line of centres.

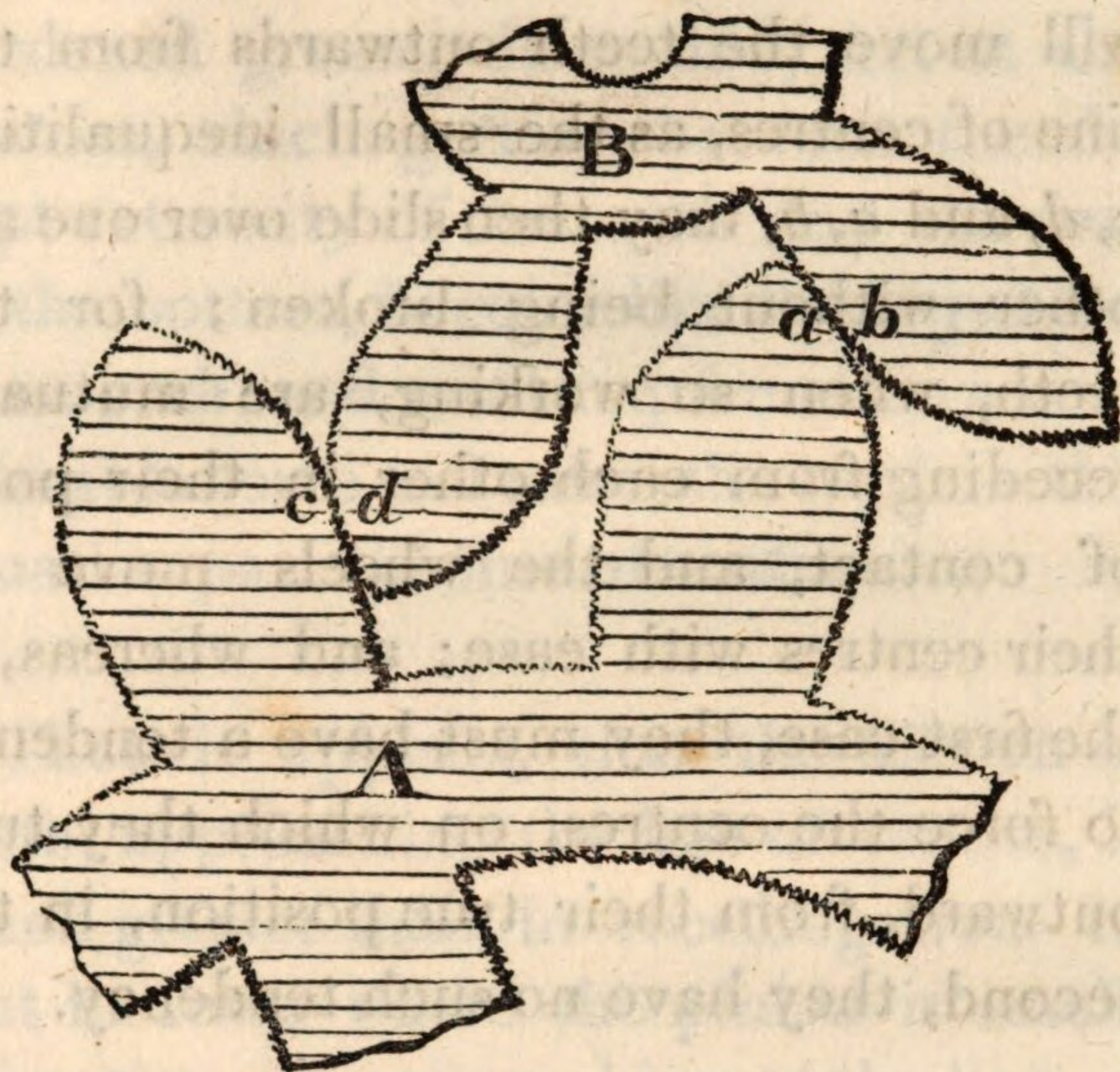
But were it so, that the extremity P, did not reach the point E, till after the flank, O N, had arrived at the line of centres; it would not be necessary, that the curved parts of the leaves, should be

pushed by the flanks of the teeth. Thus, in this case, the wheel would conduct the pinion, in pushing its leaves beyond the line of centres only.

6. It is the general opinion of those who are in the practice of constructing wheel work, that teeth ought, if possible, never to begin to act before they reach the line of centres, as that mode of action is thought to occasion much unnecessary friction. The cause of this great unnecessary friction, when the teeth are of wood, appears to be the following:

Friction depends not only upon the pressure made on moving bodies, but on the inequalities of the surfaces upon which they move; and as the surfaces even of the most highly polished bodies have some inequalities, whenever two of them are pressed together, the inequalities of the one must enter the other.

Suppose A and B to be a wheel and



pinion, having wooden teeth, as they would appear through a microscope ; it is impossible, though there be no other resistance than that arising from friction, to move them towards the line of centres, until either the centres, on which the wheels turn, give way, or some of the small inequalities, *c*, *d*, of the teeth be broken off*.

* See a further illustration of this subject in the Supplementary Observations.

On the other hand, a very small force will move the teeth outwards from the line of centres, as the small inequalities, *c, d*, and *a, b*, may then slide over one another without being broken; for the teeth, when so working, are mutually receding from each other in their point of contact, and the wheels move on their centres with ease; and whereas, in the first case, they must have a tendency to force the centres, on which they turn outward, from their true position, in the second, they have no such tendency.

Again, when the teeth are of metal, this unnecessary friction seems to arise principally after the teeth are in some degree worn (See the figure in page 58)—The teeth in that case have a kind of seat formed at their bottom, and the curve at the outward extremity is too much inclined to the radius, and very abrupt. In the action which takes place before the line of centres, the sliding of the teeth of the conducting wheel along those of the

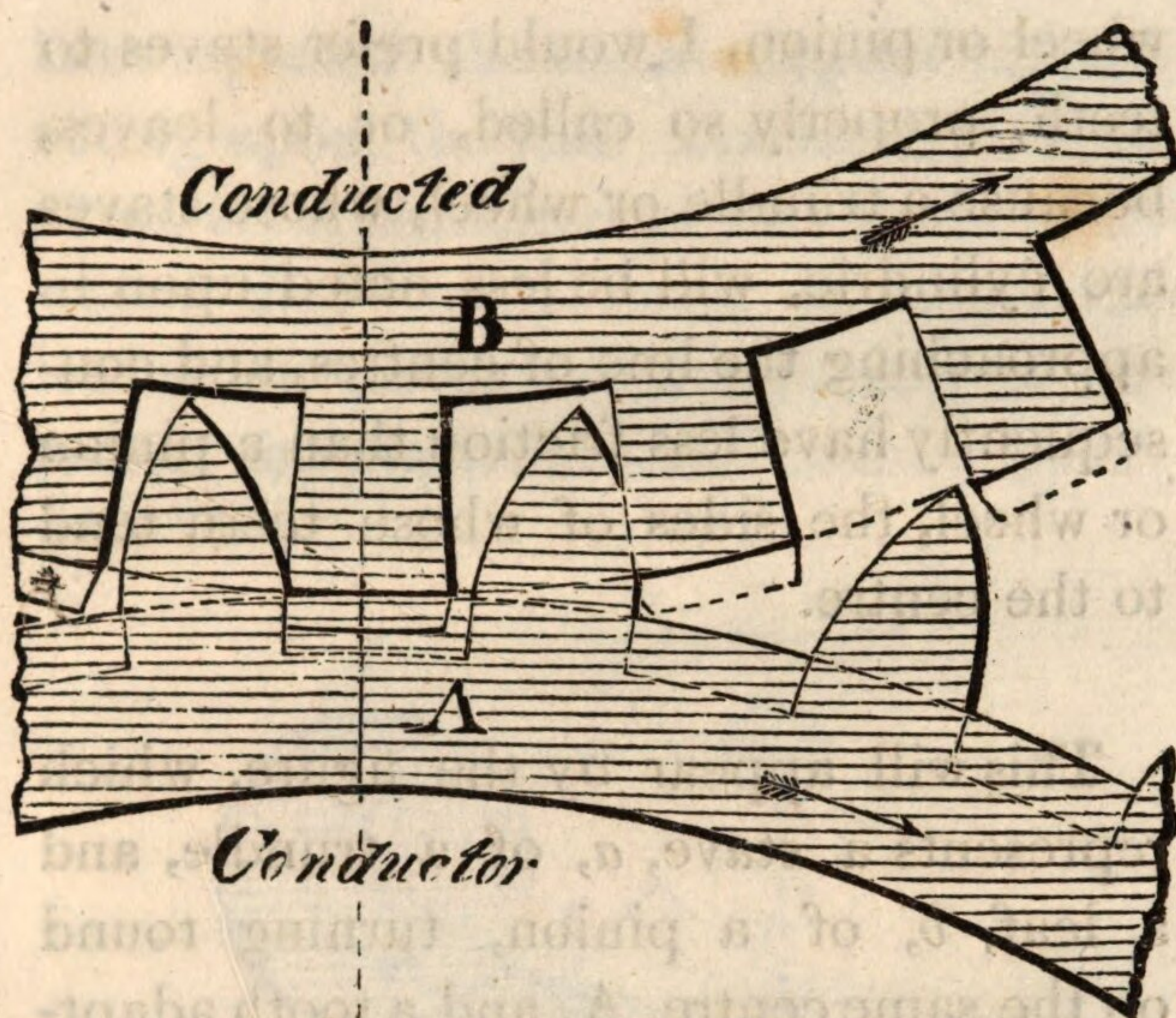
conducted, has a tendency to accumulate hardened grease, dust, sand, &c. at the bottom, which getting between the abrupt extremity of the tooth and the seat at the bottom, become like the key-stone of an arch, and must require often a considerable force to bruise the teeth in this situation past the line of centres.

Thus it appears, that the friction of teeth, approaching the line of centres, is much greater than in receding from it. But in cases where the pinion is small, the action, in approaching to the line of centres, cannot be altogether prevented. M. Camus, in his "*Cours de Mathématique*," has demonstrated, that a wheel of 50 teeth cannot conduct a pinion of 7 leaves, without their acting partly before they arrive in the line of centres. He also proves the same with regard to 57 teeth and 8 leaves, 64 and 9, 72 and 10*.

* See M. Camus.

7. From what I have already said, it will be evident, when the pinion consists of such a number of teeth, as to be conducted uniformly by the wheel in *receding* only from the line of centres, that except in small numbers, the epicycloid is necessary on the conductors only, whether it be a wheel or pinion. For instance, in p. 55, which represents two wheels of equal numbers, A is the conducting, and B the conducted wheel. But it is to be observed, when of two wheels acting on each other, sometimes the one, and sometimes the other, is the conductor, the teeth of both should be epicycloidal, as in p. 46.

When the teeth of the conducted wheel or pinion, are acted upon by those of the conductor, in *receding* only, from the line of centres, it may be remarked, if they were perfectly made, and of durable materials, it would be unnecessary to extend the conducted teeth beyond their proportional circle. But these properties



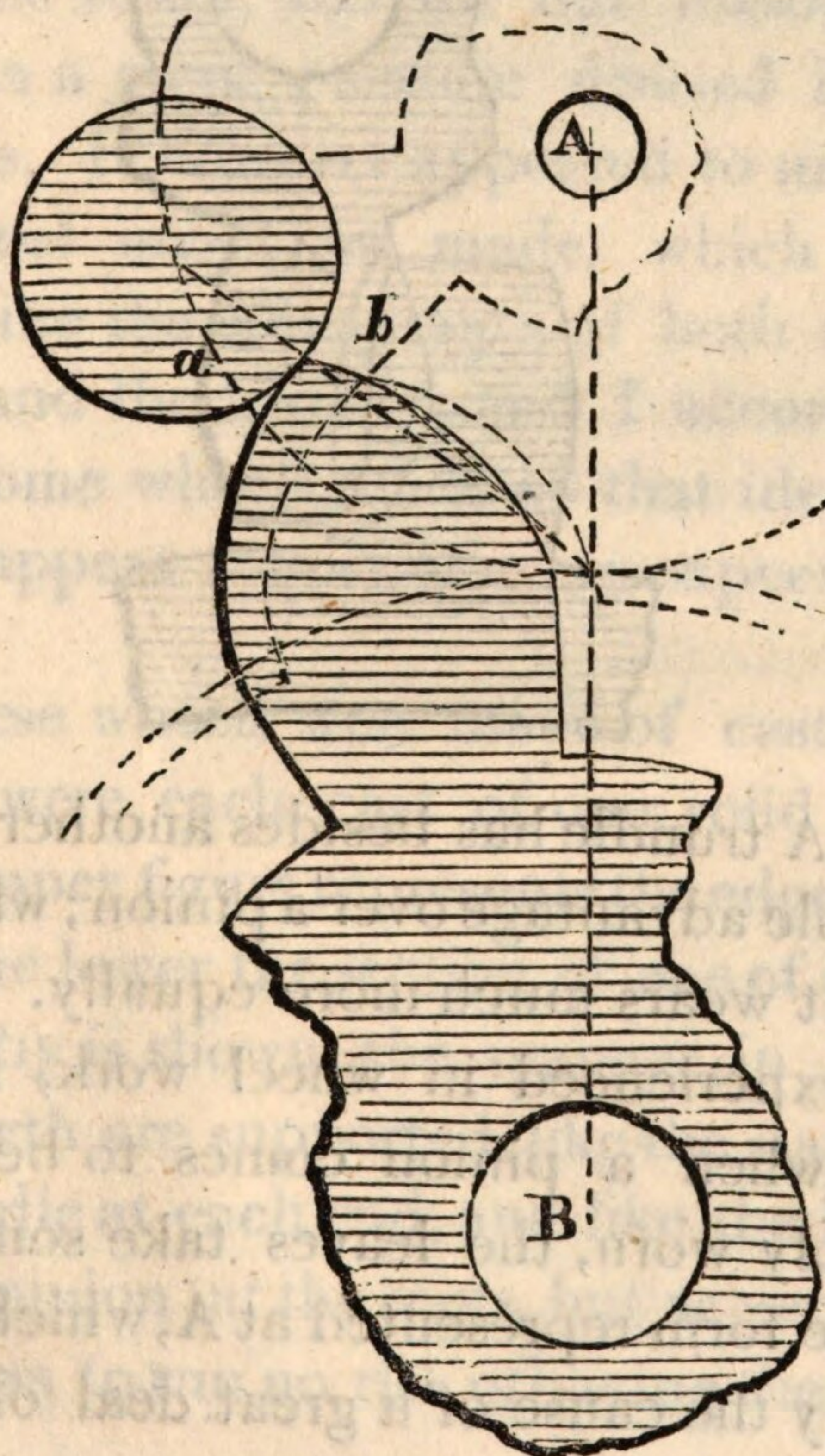
being unattainable, and as the angles which terminate their sides, would be apt to cut the conducting teeth, and occasion an irregular motion, it is proper to form the extremity of the teeth of the conductor, in the manner represented in the figure by the dotted lines.

8. Sometimes it may be requisite to have but few teeth in the pinion. In such cases, in the *conducted*, whether

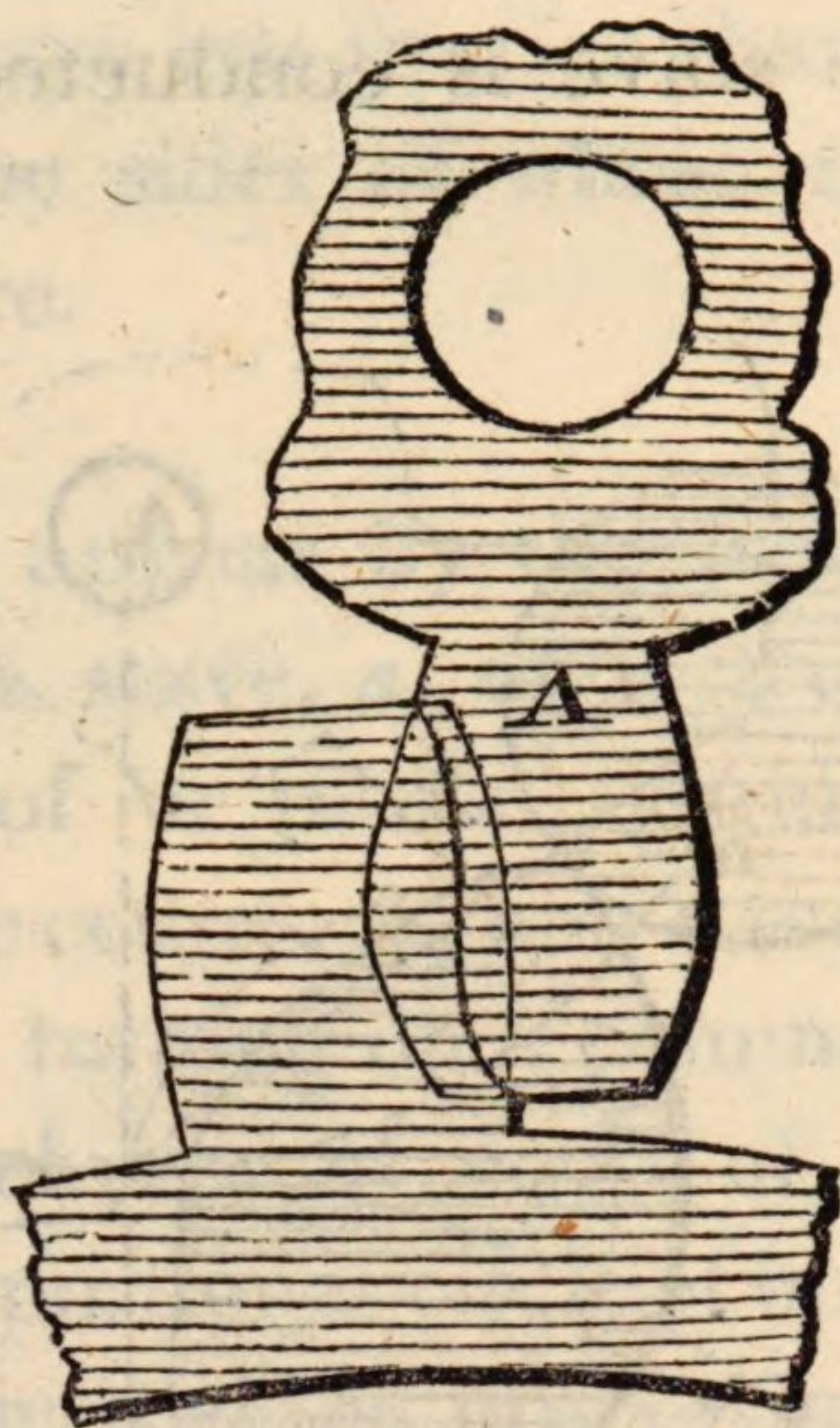
wheel or pinion, I would prefer staves to teeth, properly so called, or to leaves, because a trundle or wheel, whose staves are cylindric, will be less acted upon in approaching the line of centres, and consequently have less friction than a pinion or wheel, the sides of whose teeth tend to the centre.

This will appear by the figure, which represents a stave, *a*, of a trundle, and a leaf, *b*, of a pinion, turning round on the same centre, *A*, and a tooth adapted to each, turning on a common centre, *B*. The thickness of each of the teeth, and the proportional circle of both wheels, are the same, and the proportional circles of the pinions are also equal, and teeth are each made of the greatest length, which the intersection of the curves will admit, which turns out considerably greater in the tooth adapted to the stave. The shaded parts represent the teeth adapted to, and acting upon, the stave; and the dotted

lines represent the tooth adapted to, and acting upon, the leaf. The teeth, in both cases, are represented as just at the point where they would cease to move the leaves or staves uniformly; and it appears, the stave is conducted consider-



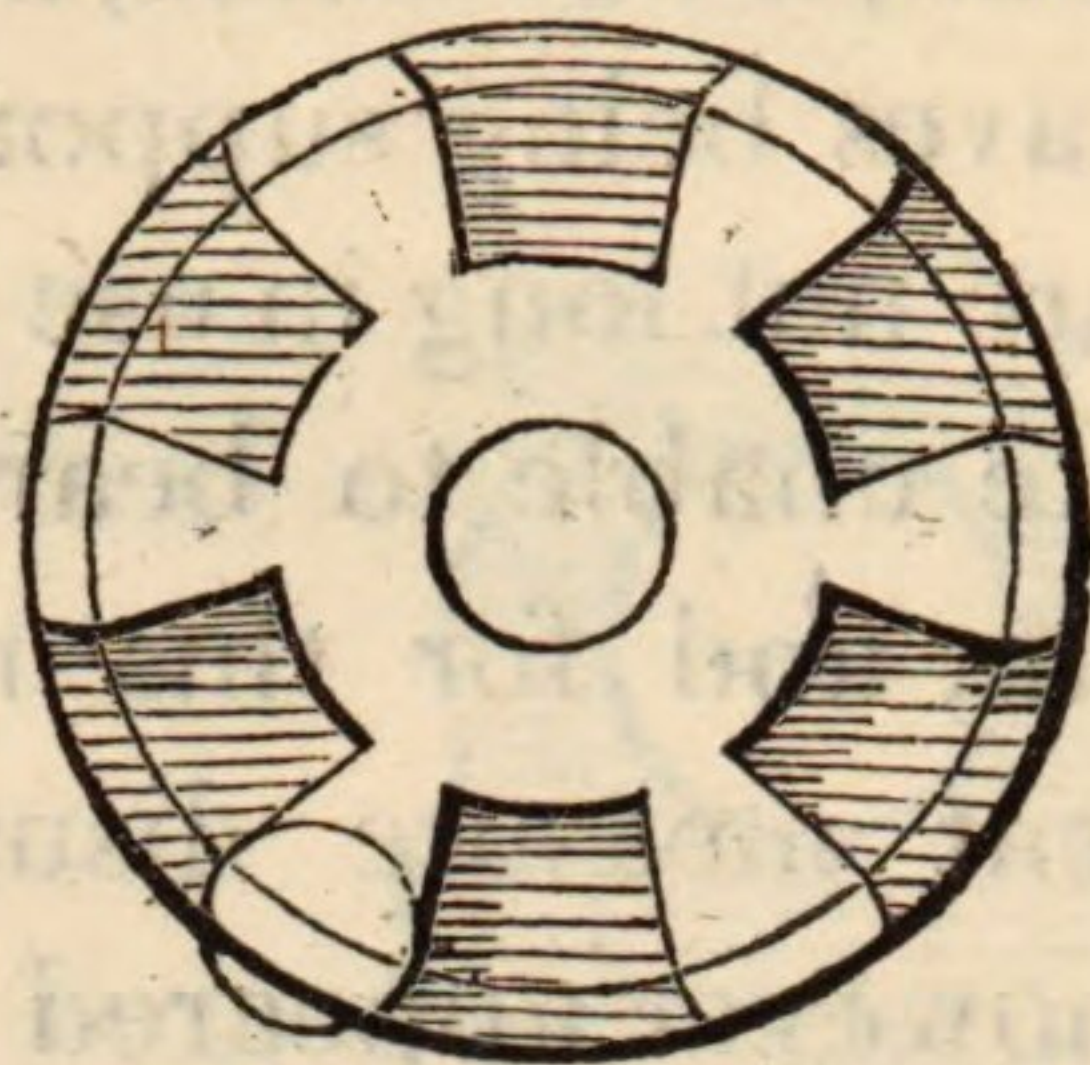
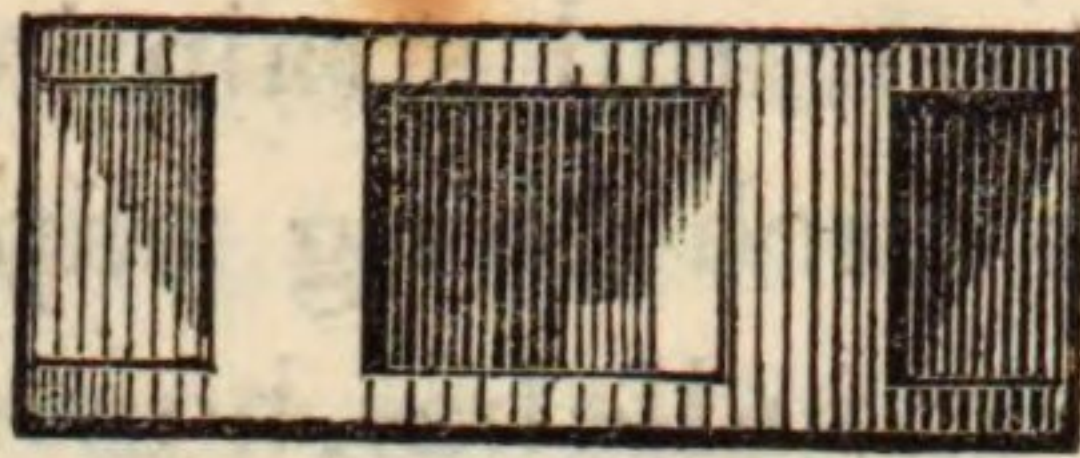
ably further beyond the line of centres than the leaf; hence the stave will be less acted upon in approaching the line of centres.



9. A trundle has besides another considerable advantage over a pinion; which is, that it wears much more equally. Every one experienced in wheel work, knows, that when a pinion comes to be considerably worn, the leaves take somewhat of the form represented at A, which is evidently the cause of a great deal of unne-

cessary friction, and strain in a machine. Whereas no such thing happens to the trundle. The trundle has, however, a defect perhaps as bad, if not worse, that of weakness. Its staves being supported at the ends only, are not long in use before they become quite unable to bear any considerable strain, and for this reason, it is now in a great measure disused in machines. It however appeared to me, that a wheel might be made, which would combine the advantages of both the pinion and the trundle, and I accordingly had some wheels made on that idea, and they appear to answer every expectation.

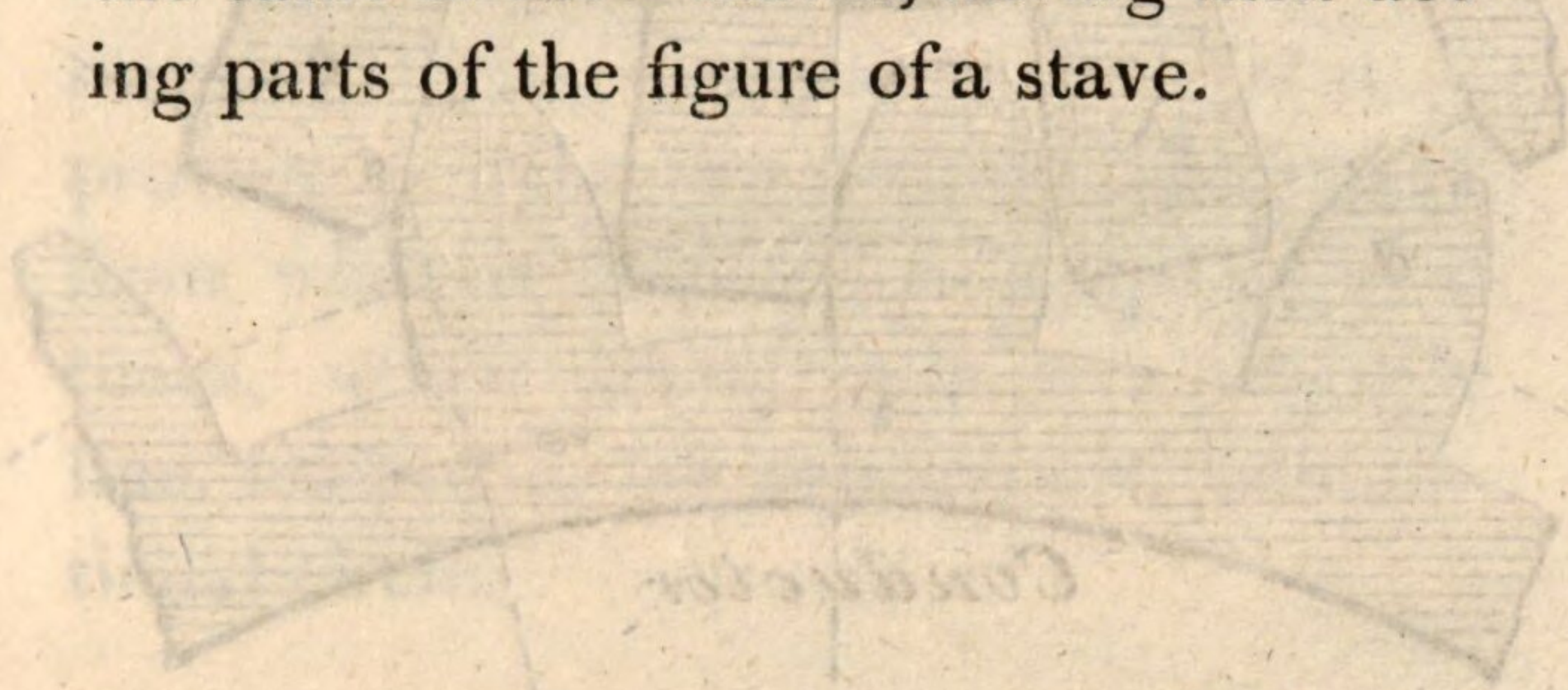
These wheels were made of cast iron. They were each cast of one solid mass. The upper figure represents the edge view, and the lower the section of one of them ; whereby is shown the manner in which the teeth are supported, like the staves of a trundle at each end, and like the leaves of a pinion at the roots, but so very thin there, as to run no risk of having the com-



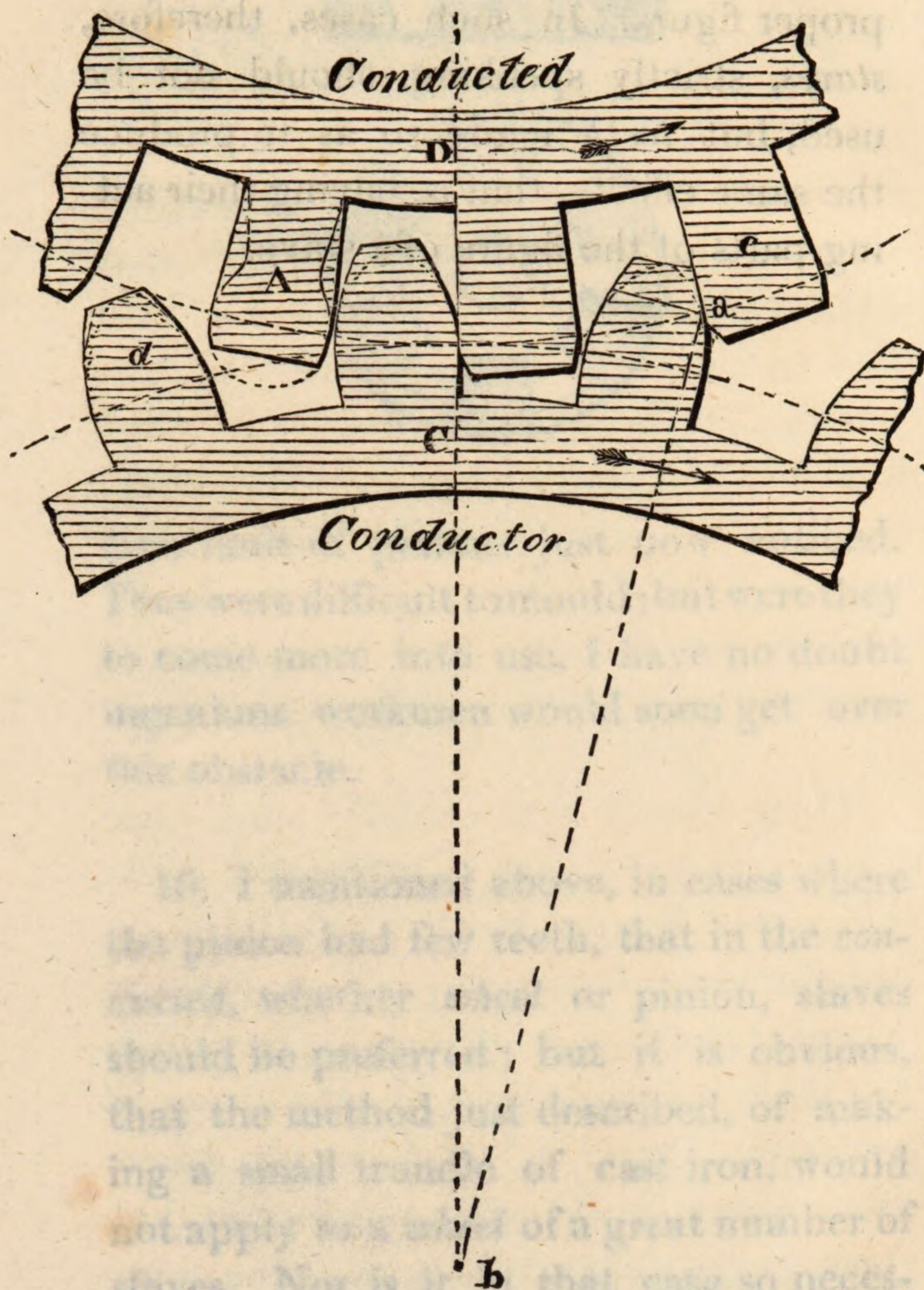
mon fault of pinions just now noticed. They were difficult to mould; but were they to come more into use, I have no doubt ingenious workmen would soon get over this obstacle.

10. I mentioned above, in cases where the pinion had few teeth, that in the *conducted*, whether *wheel* or pinion, staves should be preferred; but it is obvious, that the method just described, of making a small trundle of cast iron, would not apply to a *wheel* of a great number of staves. Nor is it in that case so neces-

sary, as the greater the number of teeth are, the longer they will be in losing their proper figure. In such cases, therefore, *staves*, strictly speaking, should not be used, but *teeth* made so as to produce the same effect—that is, having their acting parts of the figure of a stave.



What is meant will be better understood, by inspecting the figure, where



the lines show the alteration necessary on the tooth A, in order to make it produce the effect of a stave ; which stave is represented by the faint dots. The dotted lines on *d*, represent the alteration requisite to adapt it to the stave, it being necessary, as formerly proved, to have it a different epicycloid from what is required to adapt it to a tooth, whose acting part is a straight line, tending to the centre of its proportional circle.

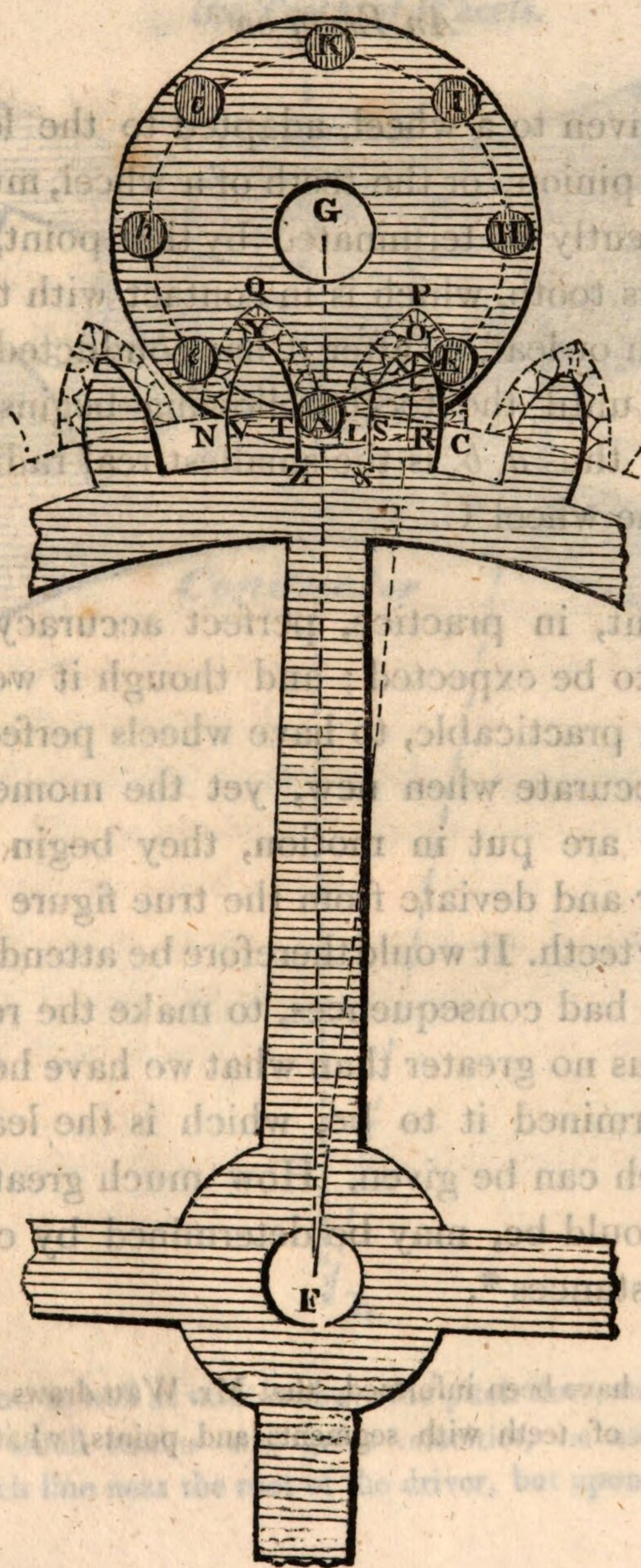
11. Hitherto the conducting teeth have been considered, as being always made as long as the epicycloidic form of their sides would admit. This however is not always necessary, and in some cases may be improper.

It now remains to show, the smallest real radius a wheel, adapted to a trundle, can have, without destroying the uniformity of the motion.

When the stave E shall have been conducted to the situation in which it is represented, the point T, of the following stave, shall be in the line of centres G F. The stave A, in its turn, may then be conducted by the following tooth, T, Y, V, and then it shall no longer be absolutely necessary, that the tooth, R, O, S, conduct the stave E. The tooth, R, O, S, may therefore be terminated in the point X, where it shall touch the stave E, when the point T, of the following stave, shall be in the line of centres, and the distance, X, F, of this touching point, from the centre of the wheel, shall be the least real radius which can be given to the wheel.

To determine the point X, draw from the centre of the stave E, to the point T, the straight line E T, and where this line meets the circumference of the stave E, you have the point required.

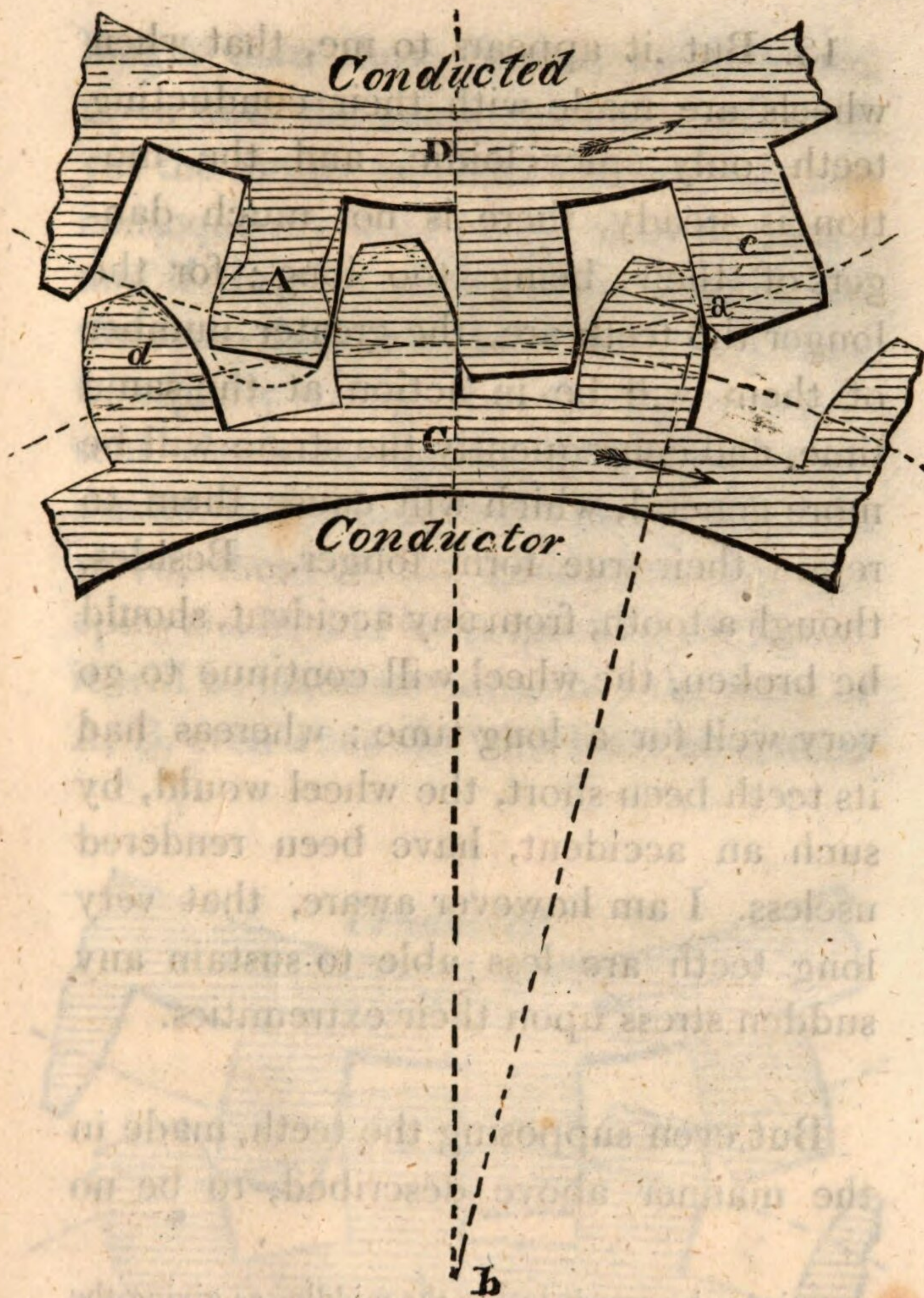
12. The smallest real radius which can



be given to a wheel, adapted to the leaf of a pinion, or the teeth of a wheel, must evidently be terminated by that point, *a*, of its tooth, which is in contact with the tooth or leaf, *e*, after it has conducted it just until the tooth following begins to act: thus *a, b*, is the smallest real radius of the wheel C.

But, in practice, perfect accuracy is not to be expected; and though it were even practicable, to have wheels perfectly accurate when new, yet the moment they are put in motion, they begin to wear and deviate from the true figure of their teeth. It would therefore be attended with bad consequences, to make the real radius no greater than what we have here determined it to be, which is the least which can be given. How much greater it should be, may be determined by circumstances*.

* I have been informed, that Mr. Watt draws the figure of teeth with segments and points, what is



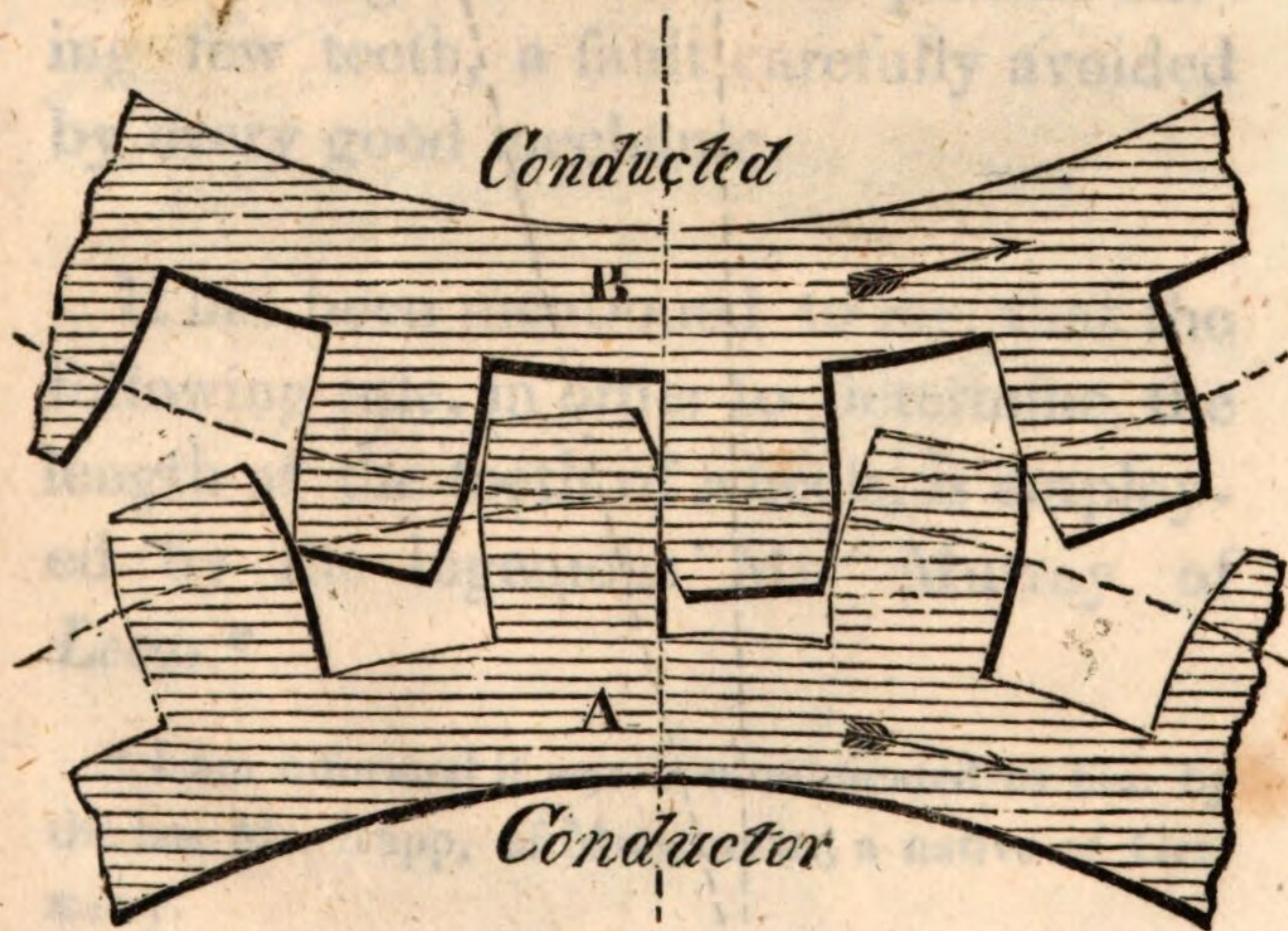
below as well as what is above the pitch lines, and that for small strains and great velocities, he uses the pitch line near the root of the driver, but upon other

13. But it appears to me, that when wheels are made with their conducting teeth only epicycloidic, and the motion is steady, there is not much danger of their being too long; for the longer the teeth are, the greater number of them will be in action at the same time, and consequently the strain will be more general, which will cause them to retain their true form longer. Besides, though a tooth, from any accident, should be broken, the wheel will continue to go very well for a long time; whereas had its teeth been short, the wheel would, by such an accident, have been rendered useless. I am however aware, that very long teeth are less able to sustain any sudden stress upon their extremities.

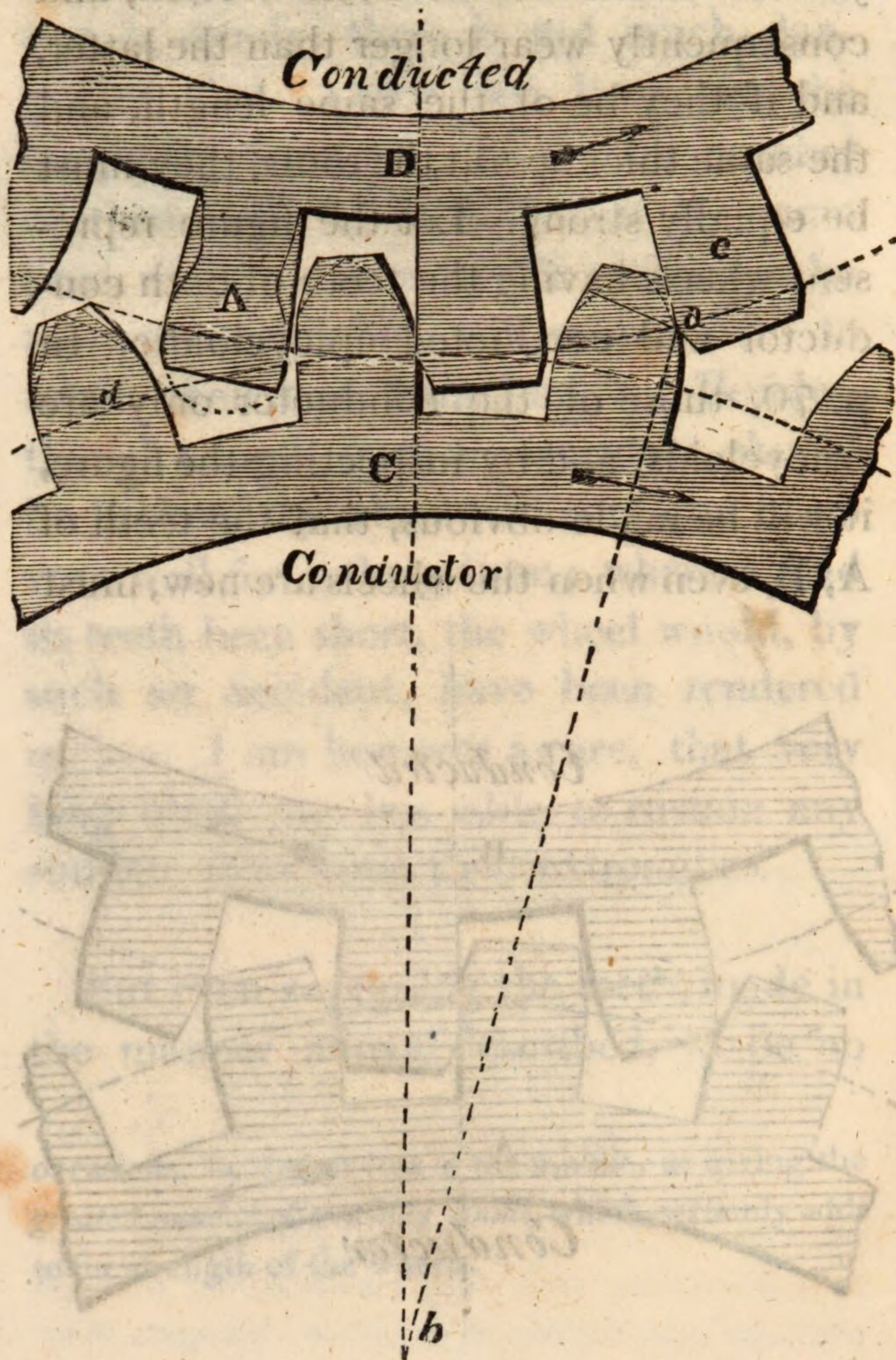
But even supposing the teeth, made in the manner above described, to be no

occasions, he uses it just in the middle, as giving the *greatest number of touching points*, which certainly adds to the strength of the wheels.

longer than those formed epicycloidic, upon both the conductor and conducted, yet the former will have less friction, and consequently wear longer than the latter, and if they be of the same length, and the same thickness at the roots, they must be equally strong. Let the figure represent wheels having the teeth of both conductor and conducted epicycloidic. In p. 70, those of the conductor only are epicycloids, and by inspecting the figure, it will be made obvious, that the teeth of A, B, even when the wheels are new, must



act as much upon each other, in approaching the line of centres, as they do



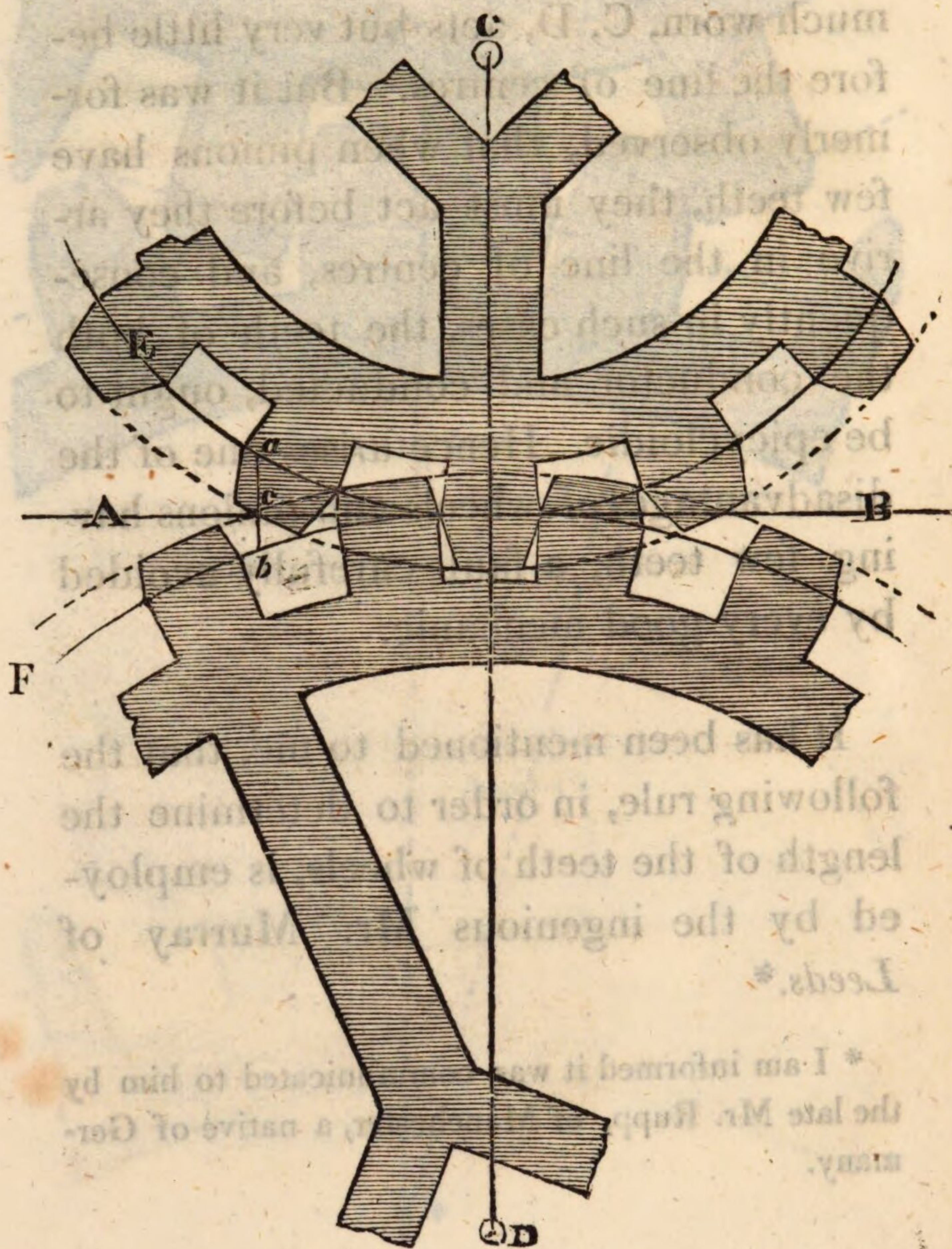
in receding from it. Whereas the teeth of C, D, when new, do not begin to act until they arrive in the line of centres; and C conducts D much further beyond that line than A does B: and even when much worn, C, D, acts but very little before the line of centres. But it was formerly observed, that when pinions have few teeth, they must act before they arrive in the line of centres, and consequently in such cases, the teeth of both the conductor and conducted, ought to be epicycloidic. Hence arises one of the disadvantages of wheels and pinions having few teeth, a fault carefully avoided by every good mechanic.

It has been mentioned to me, that the following rule, in order to determine the length of the teeth of wheels, is employed by the ingenious Mr. Murray of *Leeds*.*

* I am informed it was communicated to him by the late Mr. Rupp, of Manchester, a native of Germany.

*Rule to determine the length of the teeth of
Wheels.*

Perpendicular to the line of centres
C D, draw the line A B, a tangent to the



pitch lines. Take half the pitch, that is, half the distance between the centres of two adjoining teeth, within a pair of compasses, setting their points upon the pitch lines, E and F, parallel with the line of centres C D, draw the line *ab*, and where that is cut by the line A B, at *c*, gives the points of the teeth of wheel and pinion.

OBSERVATIONS.

On this rule, I beg leave to remark, that it does not seem to me to be founded on any satisfactory principle; were the pinion, at all times, the *conductor*, I should not perhaps differ from Mr. Murray, because the action of the teeth would be, in that case, generally *after* their arrival at the line of centres.

But in case the wheel were the conductor, the action of the teeth would generally be almost entirely *in approaching* the line of centres.

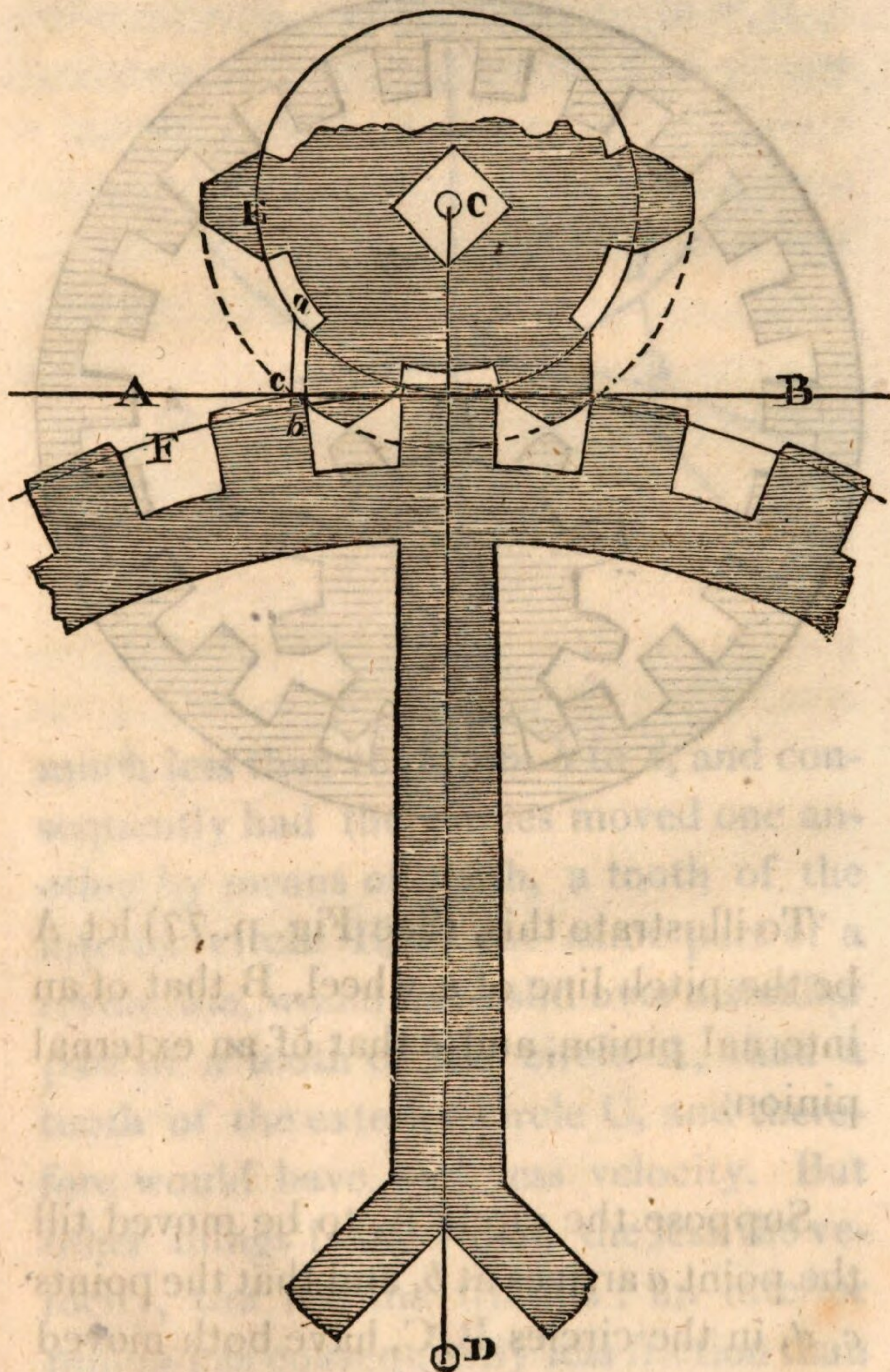
The evils arising from this mode of action, I have already, I hope, clearly proved, (see p. 50) it is therefore unnecessary here to repeat them.

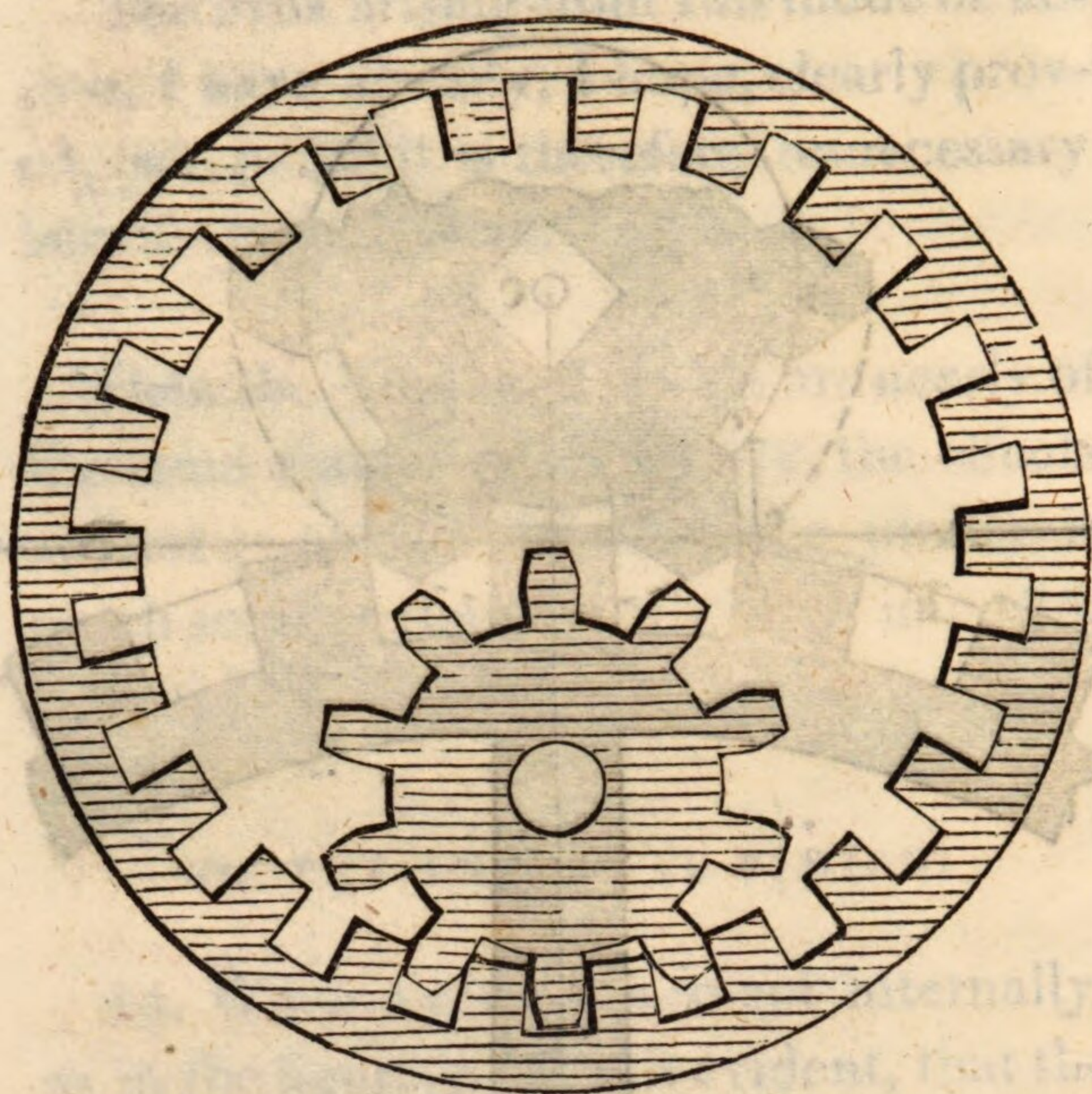
When the wheel and pinion are nearly of the same diameters, as in p. 72, the effects are not so obvious as when the pinion is much smaller than the wheel, as in p. 75.

OF THE INTERNAL PINION.

14. When a pinion is to act internally, as in the figure, p. 76, it is evident, that the teeth may be formed on the principles already laid down, with this difference only, that the epicycloid generated by the proportional circle of the pinion upon that of the wheel, should be an interior epicycloid.

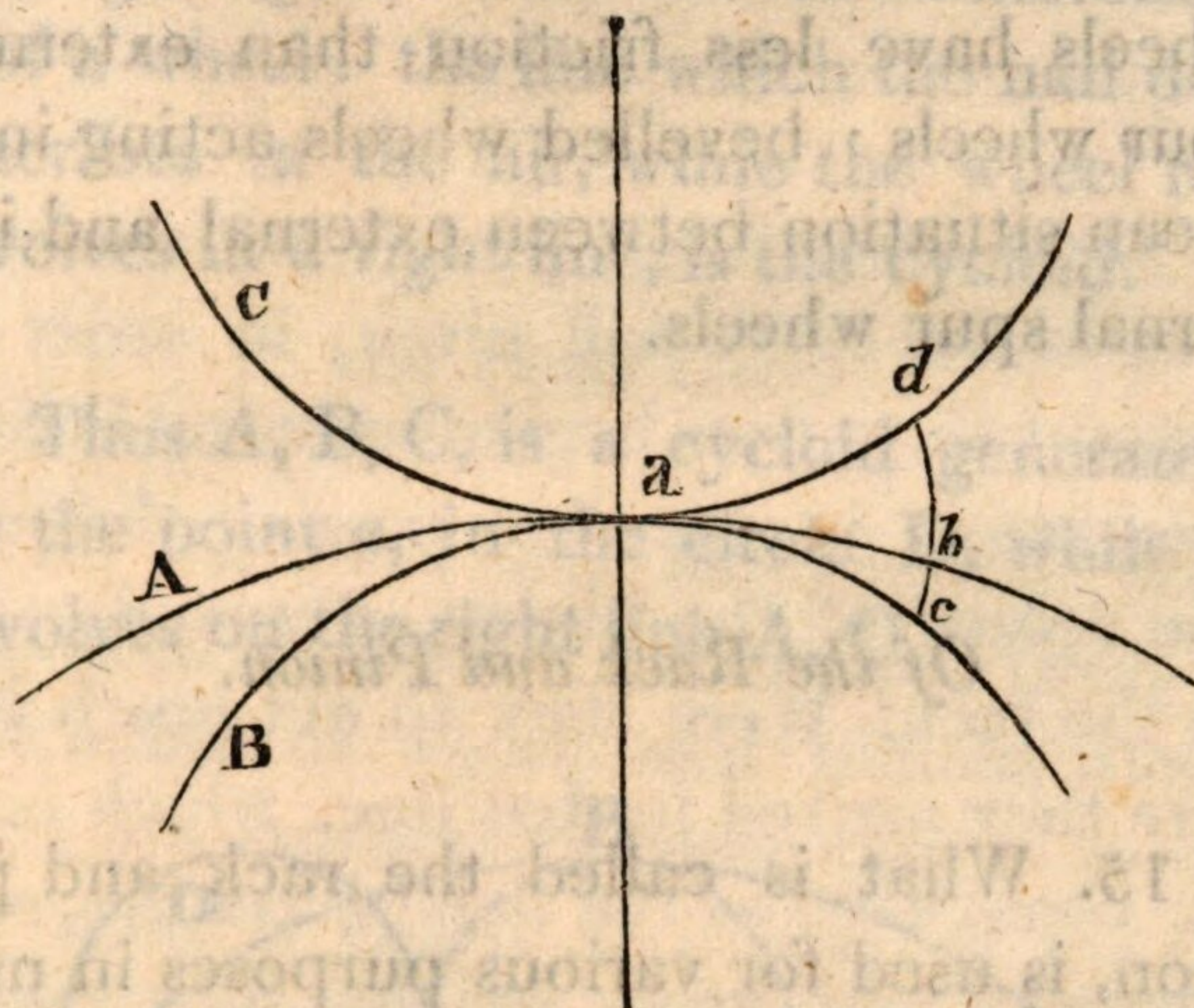
The internal pinion may be adopted in many cases with advantage, as it has less friction than the external one.





To illustrate this, (See Fig. p. 77) let *A* be the pitch line of a wheel, *B* that of an internal pinion, and *c* that of an external pinion.

Suppose the circle *A*, to be moved till the point *a* arrives at *b*, and that the points *c*, *d*, in the circles *B*, *C*, have both moved over a space equal to *a*, *b*. Now it is evident, that the distance from *c* to *b*, is



much less than that from *b* to *d*, and consequently had the circles moved one another by means of teeth, a tooth of the interior circle *B*, in the same part of a revolution, would have slid over a smaller part of a tooth of the circle *A*, than a tooth of the exterior circle *C*, and therefore would have had less velocity. But other things being equal, the less the velocity, the less the friction; an interior pinion has consequently less friction than an exterior one.

It is upon this principle, that bevelled wheels have less friction than external spur wheels ; bevelled wheels acting in a mean situation between external and internal spur wheels.

Of the Rack and Pinion.

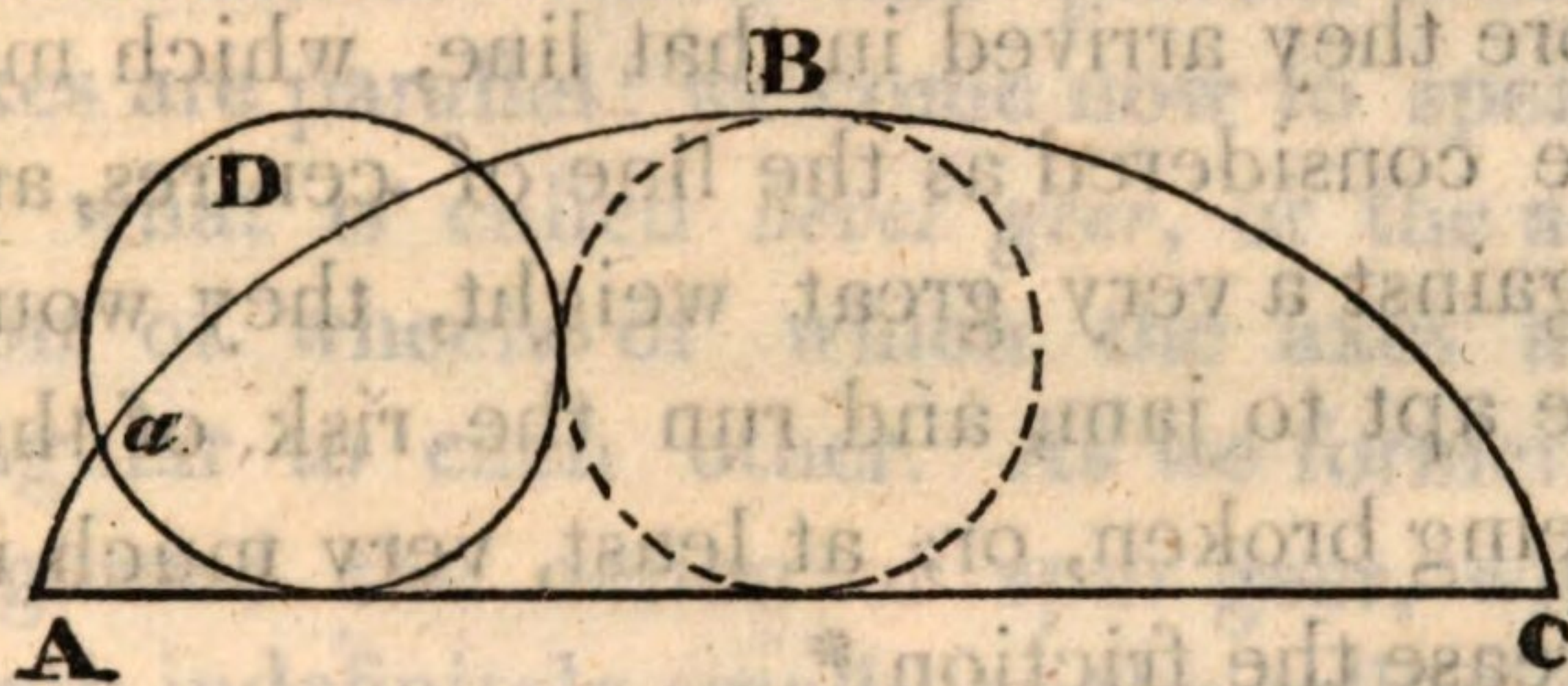
15. What is called the rack and pinion, is used for various purposes in mechanics ; as in jacks for raising great weights, and for the opening and shutting of sluices.

The rack and pinion should be made upon the principles of spur geers ; with this difference only, that in forming the teeth, the *cycloid* is, for reasons obvious from its definition, used in place of the epicycloid.

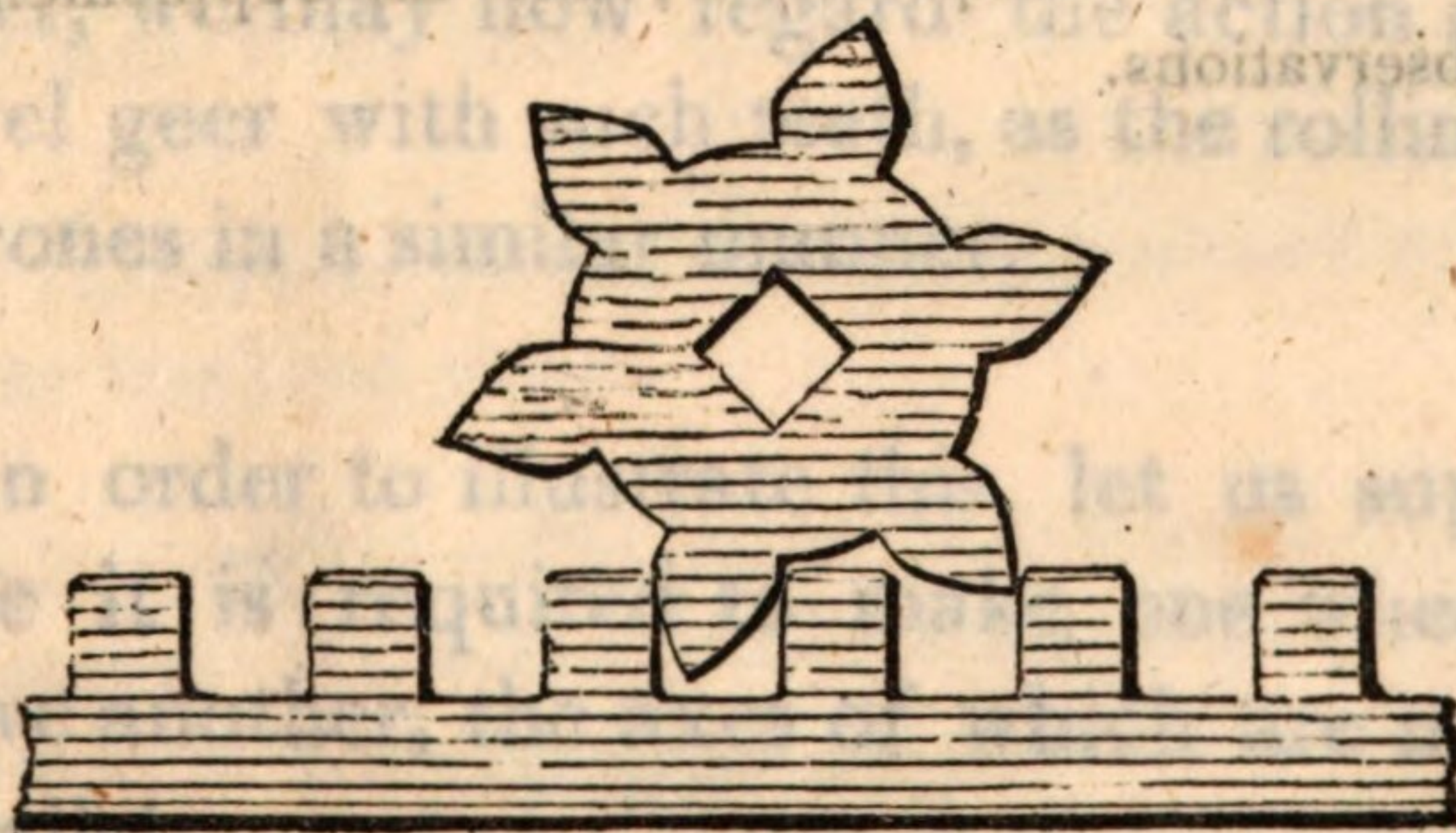
Doctor Johnson gives this definition of the cycloid : “ A geometrical curve, of

“ which the genesis may be conceived by
 “ imagining a nail in the circumference
 “ of a wheel : the line which the nail de-
 “ scribes in the air, while the wheel re-
 “ volves in a right line, is the cycloid.”

Thus A, B, C, is a cycloid generated
 by the point *a*, in the circle D, while it
 revolves on the right line A, C.



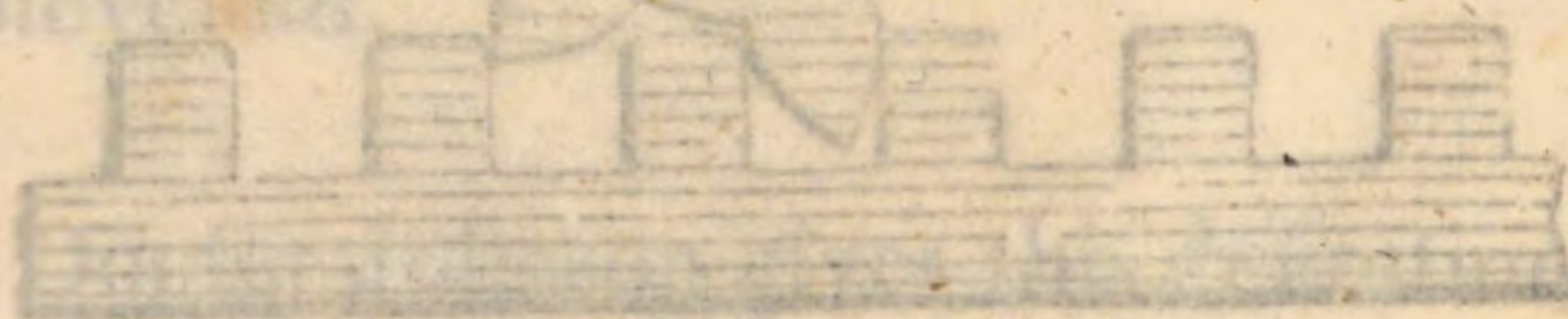
The figure represents the teeth of a rack



and pinion, formed in what seems the best mode in cases where a great weight is attached to the rack.

The leaves of the pinion are made as long as the curve will admit, in order to prevent them from beginning to act before they arrive in the line passing through the centre of the pinion, perpendicular to the rack. Were they to act much before they arrived in that line, which may be considered as the line of centres, and against a very great weight, they would be apt to jam, and run the risk of their being broken, or, at least, very much increase the friction *.

* See this Chap. No. 6; also the Supplementary Observations.

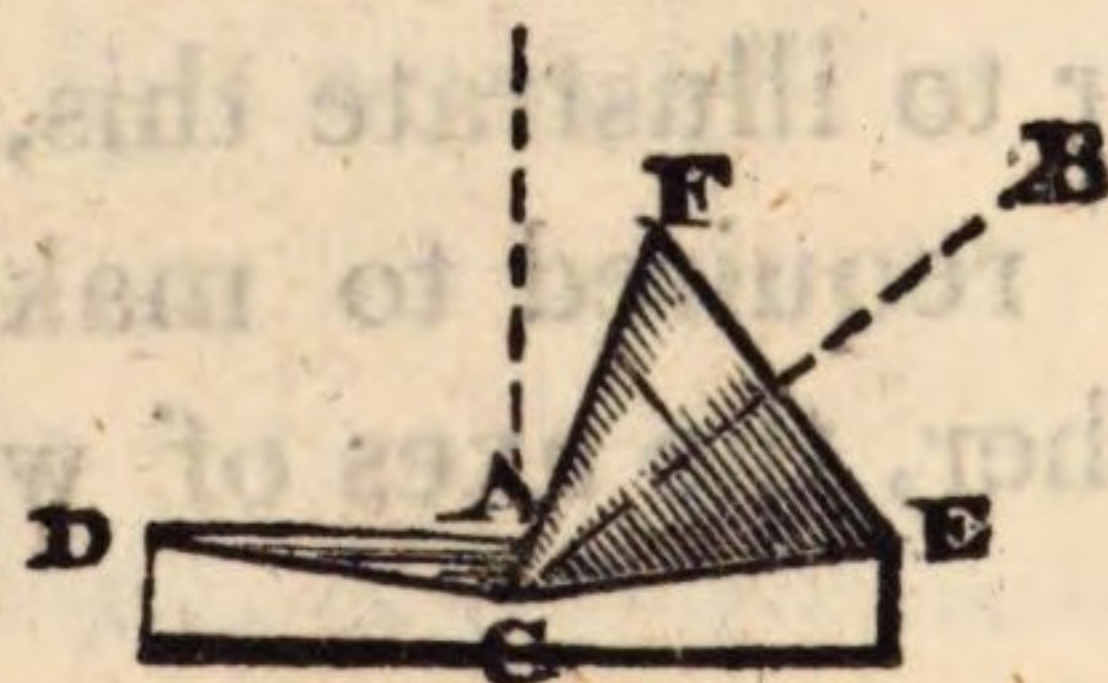
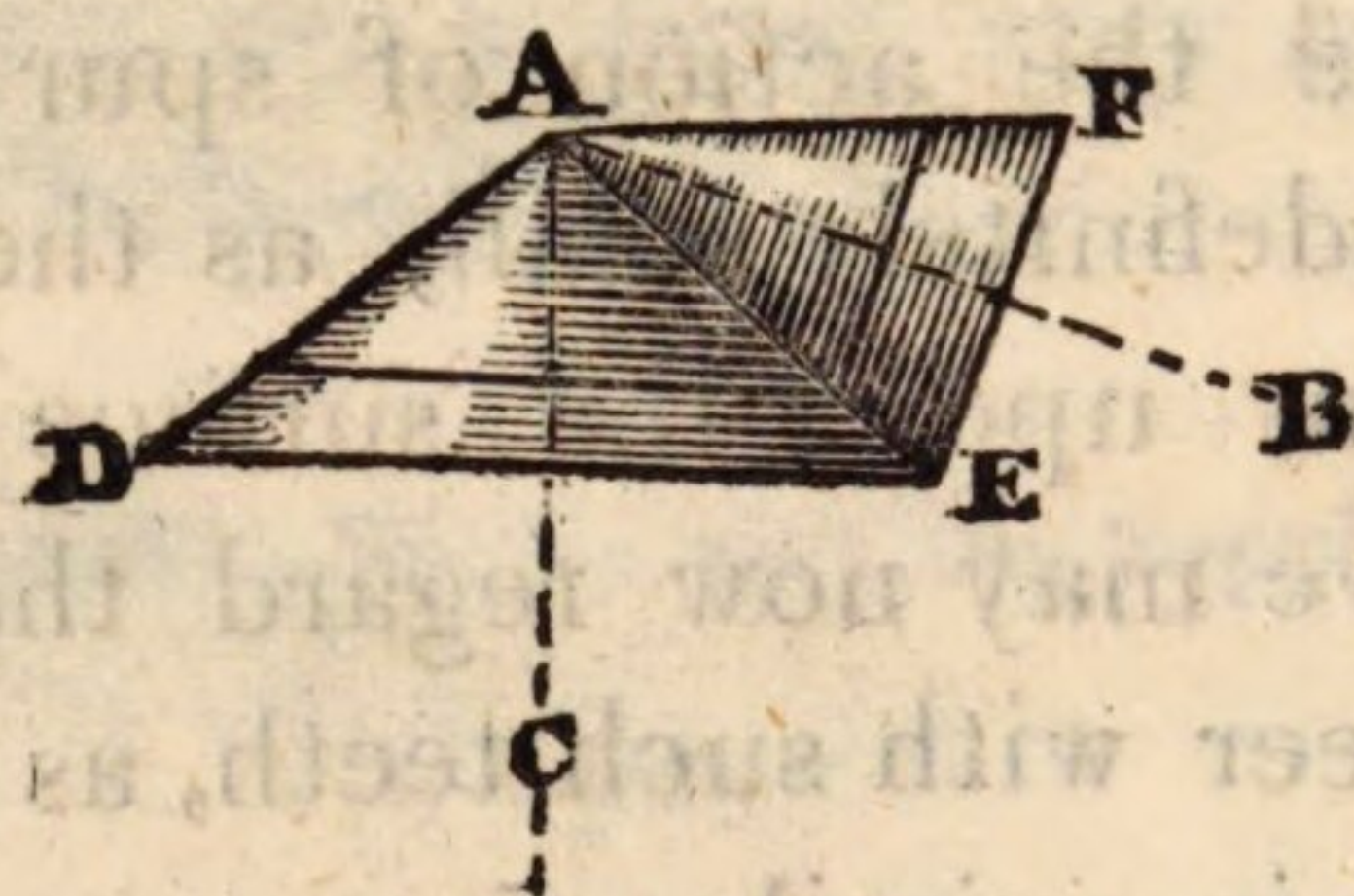
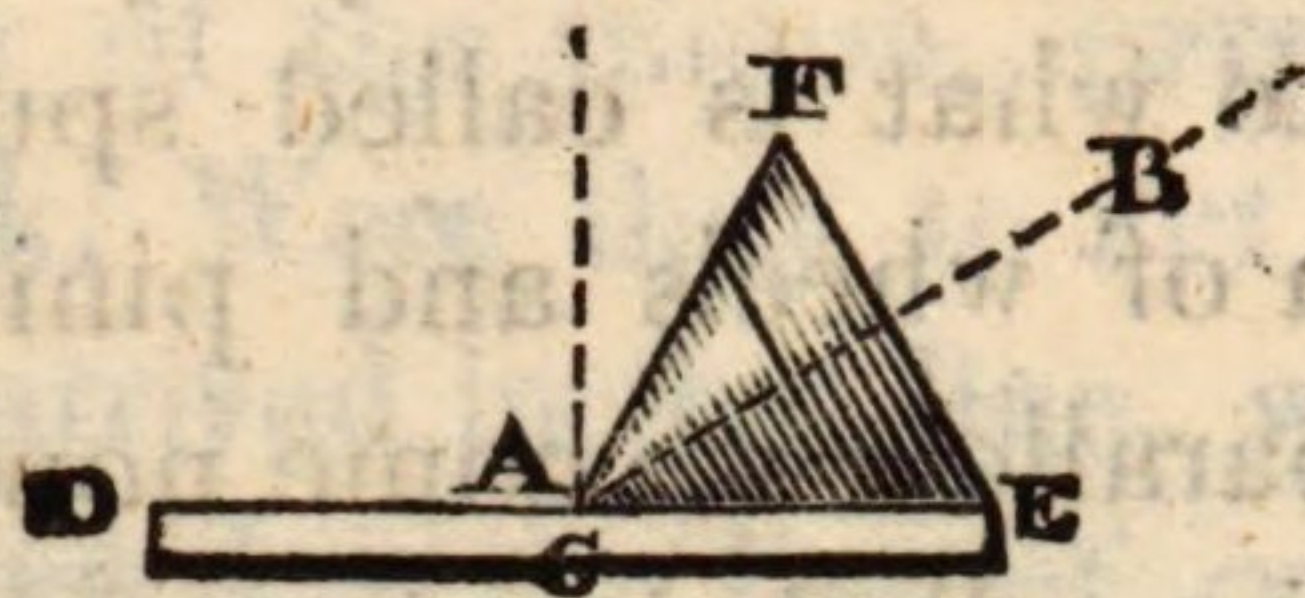
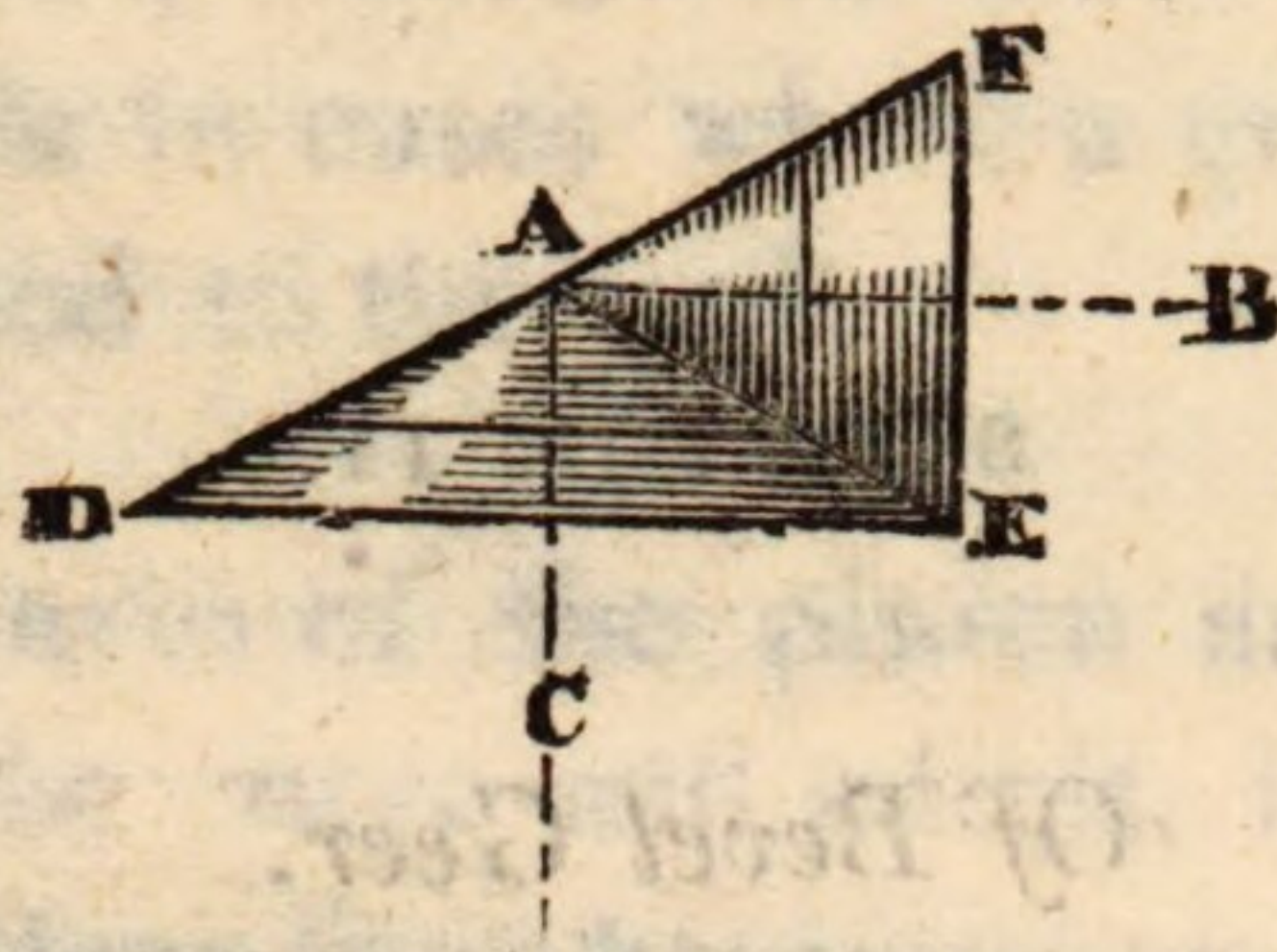


SECTION II.

Of Bevel Geer.

16. Hitherto our inquiry has been confined to what is called spur geer, or the action of wheels and pinions whose axes are parallel: we come now to speak of what is called *bevel geer*, or the action of wheels of which the axes are angular to each other. As we formerly regarded the action of spur geer, with teeth indefinitely small, as the rolling of cylinders upon the surface of each other, we may now regard the action of bevel geer with such teeth, as the rolling of cones in a similar manner.

In order to illustrate this, let us suppose it is required to make one wheel move another, the axes of which are not parallel.



Let $A B$, $A C$, be their axes, and $D E$, $E F$, their proportional diameters or pitch lines.

To the point A , where the axes intersect, draw $A E$, $A F$, $A D$; then D, A, E , and E, A, F , shall be the outline of two cones *, which rolling the one upon the surface of the other, will both revolve, so that, like two cylinders with their axes parallel †, all the corresponding points in each, shall move in every part of their revolution with equal velocity.

For, suppose any touching point D , the diameters of the cones at that point shall bear exactly the same proportion to one another, that their bases do. The same

* These cones we shall call the proportional cones of the wheel and pinion.

† It is to be observed, when the axes are in certain inclinations, the surface of one of the cones becomes concave, as in the last figure; in others, as in the second figure, the point A is in the same plane with $D E$.

may be said of every other point on their surfaces, and consequently they shall revolve in the same manner as two cylinders having their axes parallel, and the cones may be considered as bevel wheels with indefinitely small teeth.

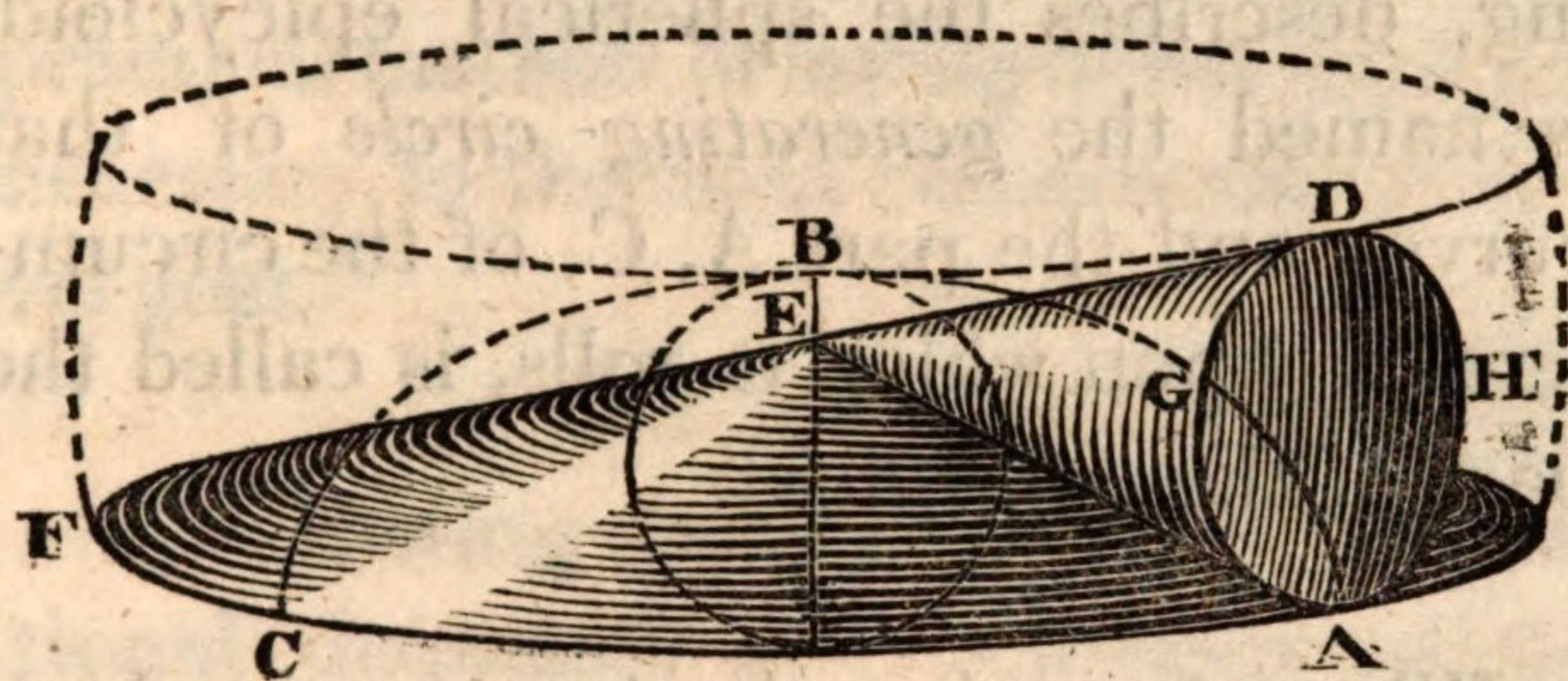
But in practice, we require finite and sensible teeth: In bevel gear, these are made similar to those of spur gear, with this difference, that in spur gear, they are parallel; but in bevel they must, as is evident, diminish in length and thickness, as they approach the summit of the cone.

The teeth may be made of any breadth, according to the strength required, and they are thereby enabled to overcome a much greater resistance, and work smoother, than is possible for a common face wheel and trundle, which, for that reason, are now superseded by bevel gear.

17. The epicycloid, which gives the

true curve to the teeth of bevel geer, differs from that used in spur geer, in being generated by the rolling of one cone upon the surface of another, while their summits coincide.

Thus for example, in the figure, the



curve, A, B, C, is described by a supposed style, fixed in the point A, of the circumference of the base of the cone A, D, E, while it rolls upon the cone F, C, A, E.

The style A, being always at the same distance from the point E, where the summit of the cone is fixed, all the points of the curve, A, B, C, shall be equidistant

from the point E, and consequently upon the surface of a sphere which shall have the point E, for its centre.

Hence the curve is called a *spherical epicycloid*.

The circle, A, G, D, H, which in rolling, describes the spherical epicycloid, is named the *generating circle* of that curve; and the part A, C, of the circumference upon which it rolls, is called the base of the epicycloid.

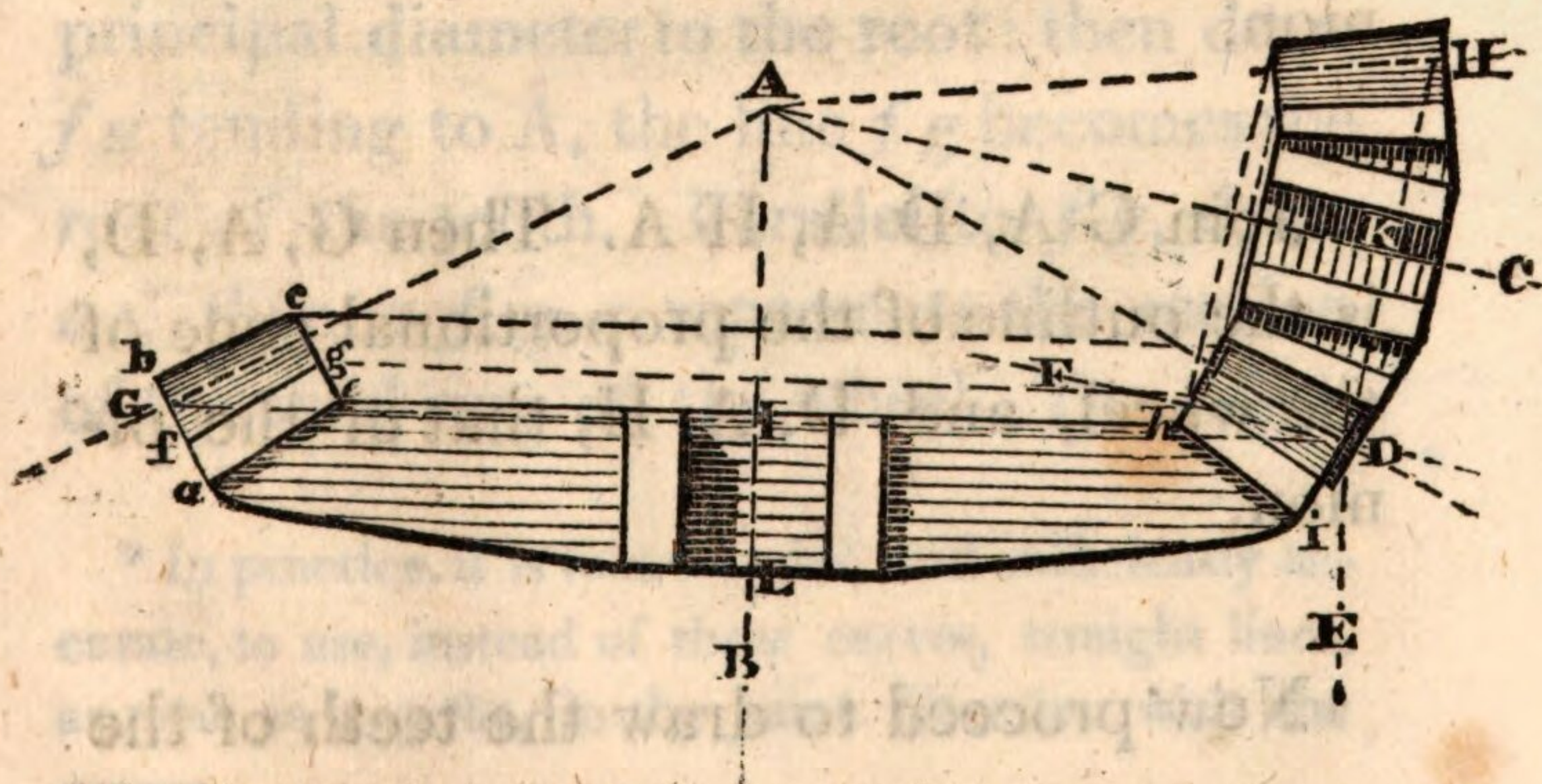
When the sphere is given upon which the spherical epicycloid is required to be traced, and we know the size and position of the rolling cone, which should generate this epicycloid, it will be easy, from what I have said relative to the plane epicycloid*, to find as many points of the curve, as may be necessary, and it will be evident, that what is said on spur

* See Chap. I. No. 5, and the note there referred to.

geer, respecting the most advantageous figure of their teeth, is all applicable to bevel geer; with this difference, that the spherical is substituted for the plane epicycloid.

In order therefore to avoid tedious repetitions, I shall immediately proceed to give some account of what seems the best practical method of laying down the lines necessary to the right construction of bevel geer.

17. Having calculated the proportional diameters or pitch lines of the



wheel and pinion, draw their axes, $A B$, $A C$, in the proposed direction with respect to each other, when the wheels are in action. Parallel to $A B$, and at the distance of half the proportional diameter of the wheel, draw the line $D E$. In the same manner, draw $F D$ at the distance of half the proportional diameter of the pinion from $A C$. From the point D , where these lines intersect, draw the line $D G$, perpendicular to $A B$, and also the line $D H$, perpendicular to $C A$. Make $G I$ equal to $I D$, and $K H$ equal to $K D$. Then $D G$ is what we shall call the *principal diameter*, or the *diameter at the pitch line* of the wheel, and $D H$ that of the pinion.

Join $G A$, $D A$, $H A$. Then G, A, D , is the outline of the proportional cone of the wheel, and D, A, H , that of the pinion.

Now proceed to draw the teeth of the

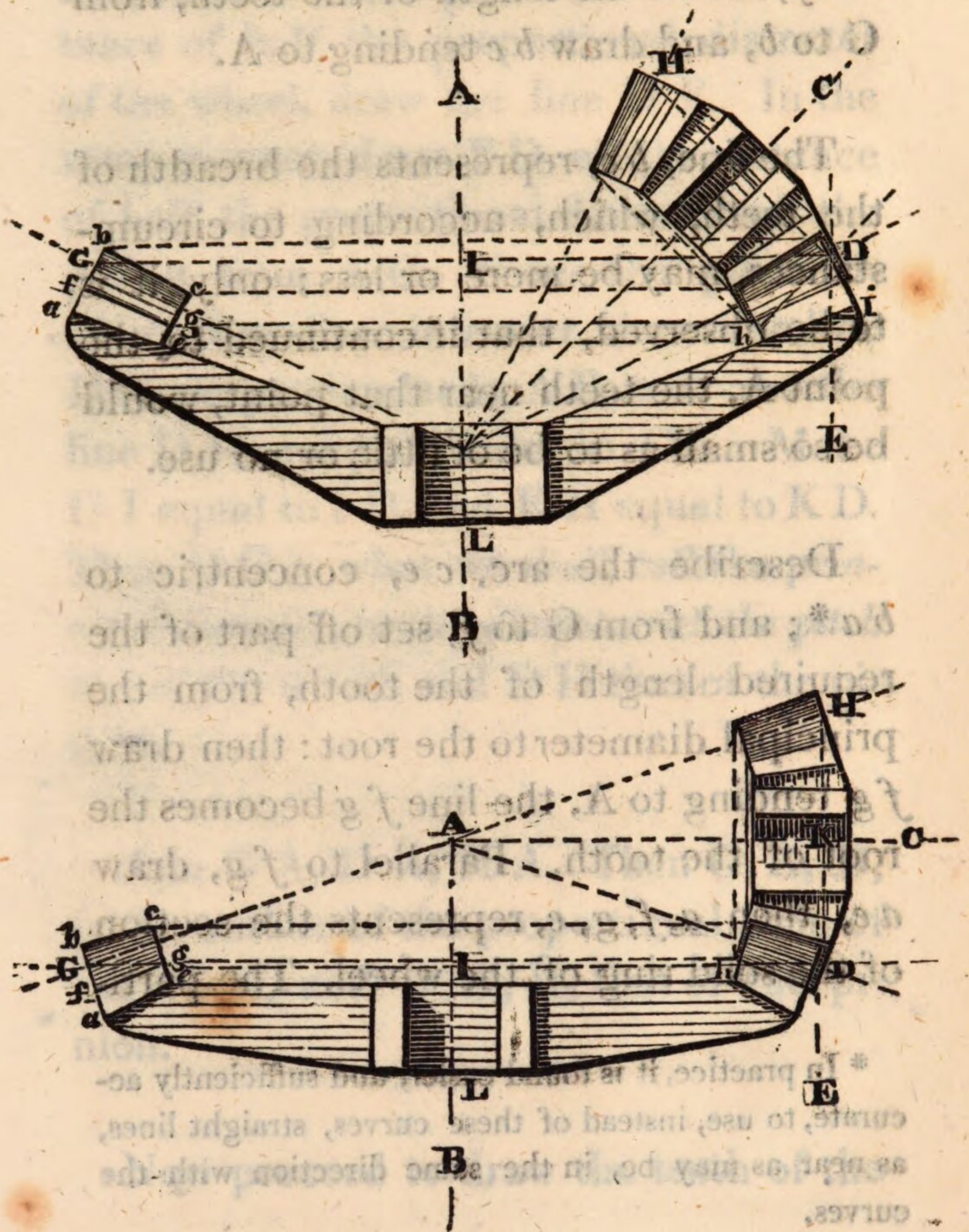
wheel. With the distance, GA , from A , as a centre, sweep a small arc, such as Ga ; at the principal diameter to their extremity, set off the length of the teeth, from G to b , and draw bc tending to A .

The line, bc , represents the breadth of the teeth, which, according to circumstances, may be more or less; only it is to be observed, that if continued to the point A , the teeth near that point, would be so small as to be of little or no use.

Describe the arc, ce , concentric to ba^* ; and from G to f , set off part of the required length of the tooth, from the principal diameter to the root: then draw fg tending to A , the line fg becomes the root of the tooth. Parallel to fg , draw ae , then a, f, g, e , represents the section of the solid ring of the wheel. The parti-

* In practice, it is found easier, and sufficiently accurate, to use, instead of these curves, straight lines, as near as may be, in the same direction with the curves.

cular direction of the line *ae*, is no way essential; all that is necessary is, that the ring be of sufficient strength for the pur-



pose to which it is to be applied : but patterns for cast iron wheels are usually made as represented in the plate.

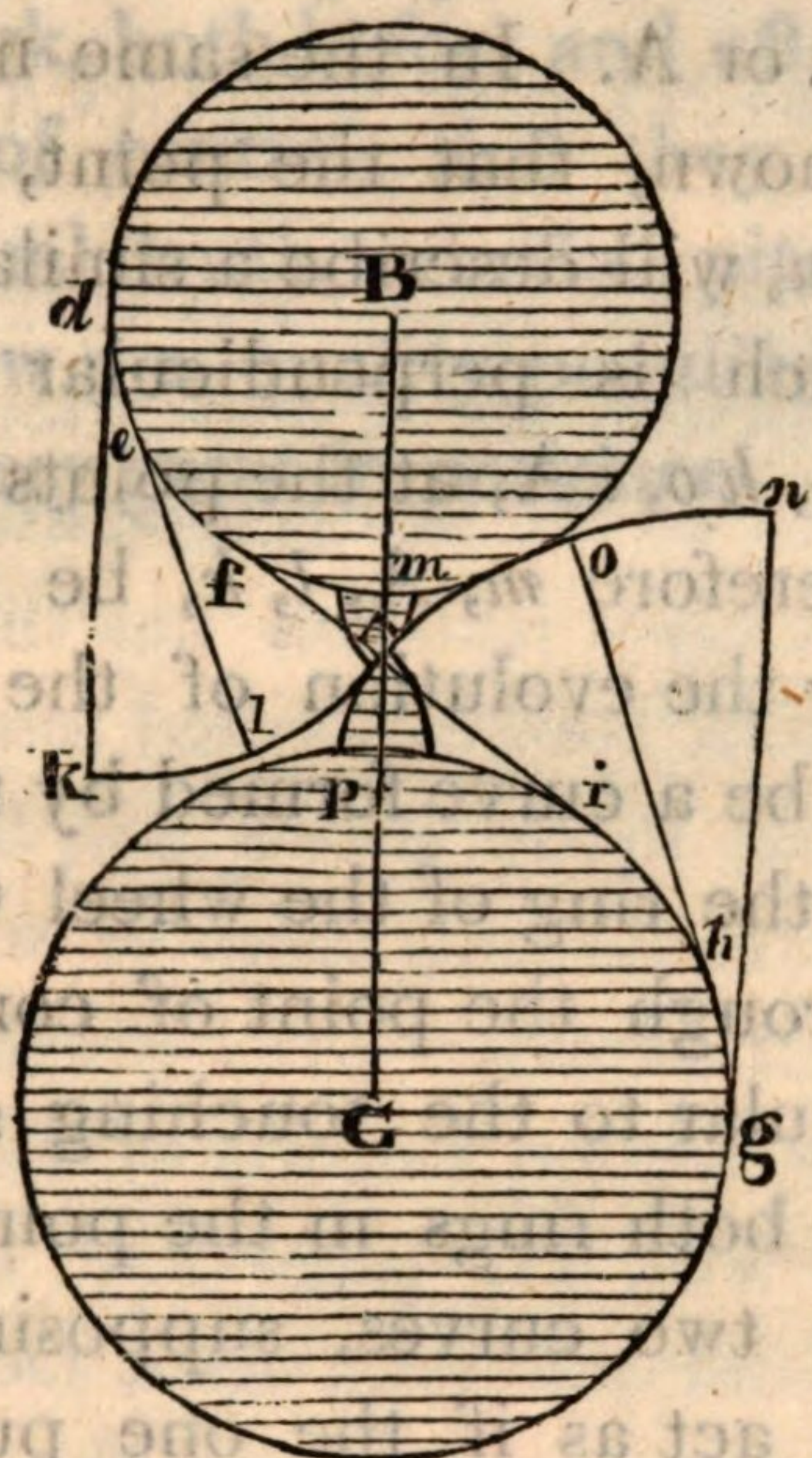
Having thus drawn a section of a tooth at G, draw in the same manner one at D, then D, I, G, L, will be a section of the wheel, in which *e, h, i, L, a*, represent the space occupied by the arms. The dimensions of these, and their particular form, may however be varied according to circumstances.

The mode of drawing the section of this pinion will now be obvious, by inspecting the figure, where it will be observed, that the teeth of the pinion are made a little broader than those of the wheel. This is a practice generally followed, as the teeth by this means wear more equally than otherwise they would.

CHAPTER III.

I shall now proceed to describe a mode of forming the teeth of spur wheels, the first hint of which, I have been informed, was given by Professor Robison, of Edinburgh. In order to understand the description and demonstration, it will be necessary to recollect what was proved in chapter I. No. 2, viz. That if a wheel move uniformly, it is necessary, in order to move another wheel uniformly, that the form of the teeth be such as that the perpendicular from the touching surfaces *in all situations*, cut the line of centres, in the same point, A, which point divides the line of centres, so that the one part, A, B, shall be to the other, A, G, as the number of teeth in the one wheel, B, is to the number of teeth in the other, G.

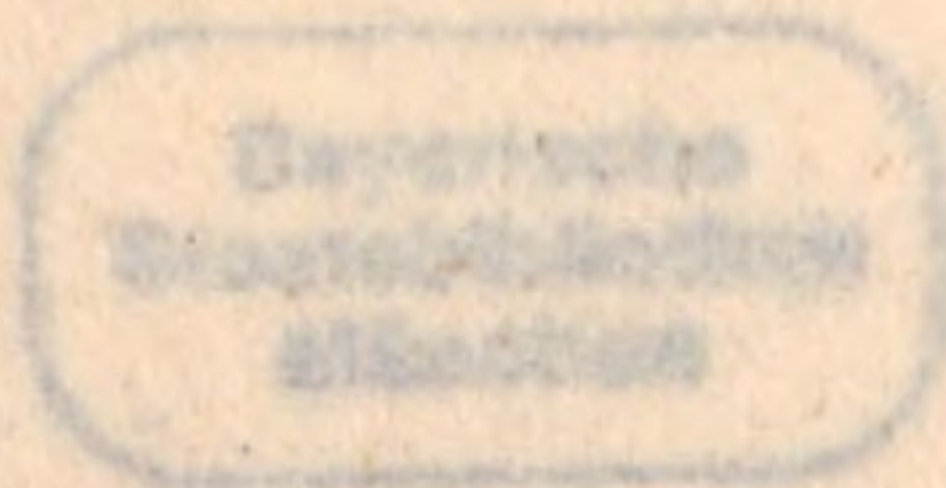
This being understood, let B and G be



the centres of the two wheels, and $d e f$, $g h i$, the rings upon which the teeth are placed ; the diameters of which rings are, to one another, as their number of teeth. If the thread, $k d$, be lapped round the ring ; as it folds up, its extremity, k , will describe the curve k, l, m . It is evident, that the thread, in describing the curve, is perpendicular to it ; and the thread, e ,

l, f, A , is therefore perpendicular to the curve at l , or A . In the same manner it may be shown, that the point, n , of the thread, $g n$, will describe a similar curve, $n o p$, which is perpendicular to the thread $g n, h o, i A$, at the points n, o and A . If therefore m, A, l, k , be a curve formed by the evolution of the ring B , and $n o p$, be a curve formed by the evolutions of the ring of the wheel G , a line drawn through the point of contact A , perpendicular to the touching surfaces, will touch both rings in the points f and i , and the two curves, supposing them teeth, will act as if the one pulled the other by the thread $i f$. The line, $i f$, will be the line of action; that is, the teeth will always touch each other in a point of this line, and since this line always passes through the same point of the line of centres, $B G$, the action will be invariable; so that if the one move uniformly, the other will also move uniformly, and two weights which balance in *any one position*, will balance in all positions of the teeth.

It is obvious, that though these teeth must work, both before and after passing the line of centres, that they will work with equal truth, whether pitched deep or shallow ; a quality peculiar to them, and of very great importance.



SUPPLEMENTARY OBSERVATIONS.

The foregoing Essay was written several years before the publication of the Supplement to the Encyclopædia Britannica.—Professor Robison has there (Vol. II. page 103, 106) described and recommended the mode which will be found, Chap. III. of forming teeth of wheels by involutes of circles. Dr. Brewster, however, in his second Edition of Ferguson's Lectures, Vol. II. page 227, observes, that this principle is not new ; De la Hire having long ago considered the involute of a circle, as the last of the exterior epicycloids ; which it may be proved to be, if we consider the generating straight line as a curve of infinite radius.

Professor Robison says,* that “ this form of teeth admits of several teeth to

* Encyclopædia Britannica, Volume XX. page 104.
See also Rees's Cyclopædia, art. Clock Movement.

be acting at the same time, (twice the number that can be admitted in M. De la Hire's method.) This, by dividing the pressure among several teeth, diminishes its quantity on any one of them, and therefore diminishes the dents or impressions which they unavoidably make on each other. It is not altogether free from sliding and friction, but the whole of it can hardly be said to be sensible. The whole slide of a tooth, three inches long, belonging to a wheel of ten feet diameter, does not amount to $\frac{1}{60}$ th of an inch, a quantity altogether insignificant.

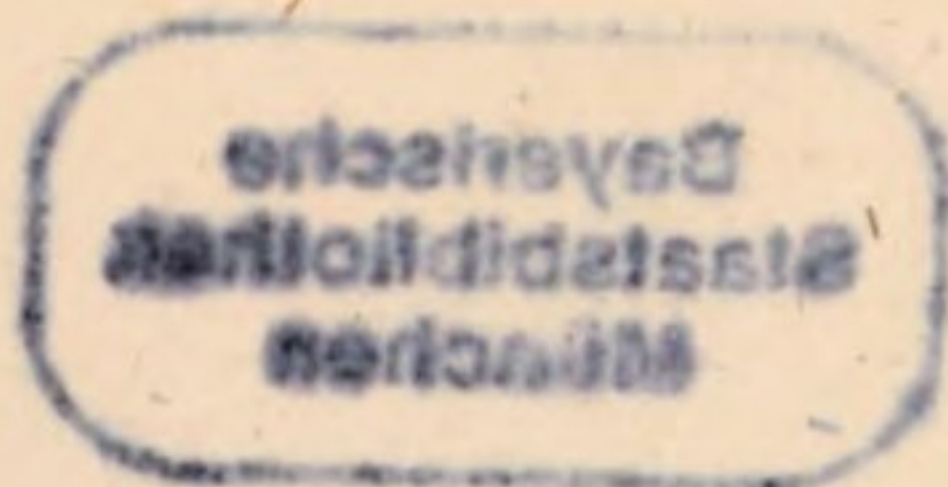
In the same article, this highly respectable philosopher was mistaken, in supposing, with other eminent authors, that *the mutual action of the teeth*, (when formed into epicycloids, by the method of M. Camus,) *is absolutely without friction*, and in saying, “*That one tooth only APPLIES itself to the other, and ROLLS on it, but does not SLIDE or RUB on it in the smallest*

N

degree. This makes them last long, or rather, does not allow them to wear." A very slight examination of the figures given in various parts of the preceding Essay, will, I hope, show, that the point of contact must slide from the pitch line of the conducting tooth outwards. Dr. Young, in his Natural Philosophy, Vol. II. page 183, says, that "*a form [of teeth,] without friction, is perfectly impracticable, although, for a single tooth, possible.*"

In the first volume of the same work, he makes the following judicious observations on our present subject:

"It has been supposed by some of the best authors that the epicycloidal tooth has also the advantage of completely avoiding friction; this is however by no means true, and it is even impracticable to invent any form for the teeth of a wheel, which will enable them to act on other teeth without friction."



“In order to diminish it as much as possible, the teeth must be as small and as numerous as is consistent with strength and durability ; for the effect of friction always increases with the distance of the point of contact from the line joining the centres of the wheels.

“ In calculating the quantity of the friction, the velocity with which the parts slide over each other has generally been taken for its measure : this is a slight inaccuracy of conception, for, as we have already seen, the actual resistance is not at all increased by increasing the relative velocity ; but the effect of that resistance, in retarding the motion of the wheels, may be shown, from the general laws of mechanics, to be proportional to the relative velocity thus ascertained. When it is possible to make one wheel act on teeth fixed in the concave surface of another, the friction may be thus diminished in the proportion of the difference of the diameters to their sum.

“If the face of the teeth, where they are in contact, is too much inclined to the radius, their mutual friction is not much affected, but a great pressure on their axes is produced; and this occasions a strain on the machinery, as well as an increase of the friction on the axes.”*

The concluding part of these observations appears to me peculiarly applicable to the figure of teeth described in our last chapter; for in wearing, they will be more liable than many other forms, to have *the face of the teeth, where they are in contact, too much inclined to the radius.*

* Young's Lectures, Vol. I. p. 176.

*Remarks on the Friction of Wheel Work, and
on the Forms best suited for Teeth.*

In a Letter from DR. YOUNG.

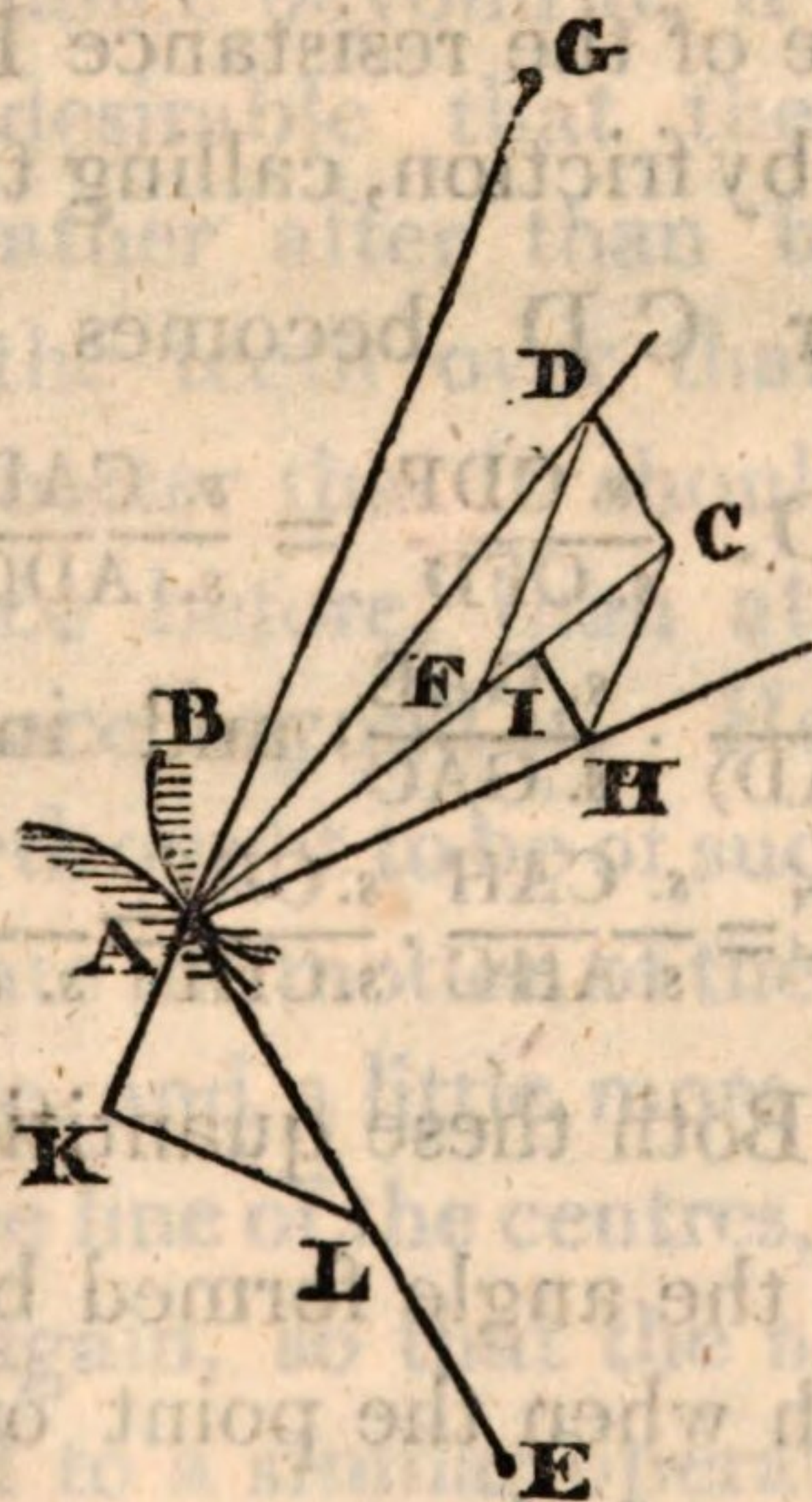
“ I have been considering your observations on the difference of the friction, accordingly as the teeth touch before or after the line of centres; at first I was disposed to doubt of the fact; but upon more mature examination, I found that, like many other practical observations, they went beyond the scope of the doctrines of theoretical writers. I cannot however perfectly agree with you as to the explanation of the fact; but I will state to you briefly my opinion on the subject, not having leisure at present to enter into a more ample discussion.

“ The magnitude of the friction has usually been estimated by the relative velocity of the surfaces concerned; a mode of calculation, which, as I have

observed in my lecture on machinery, is only so far correct, as it shows the comparative effect of a given friction in retarding the machine. But in fact the primitive friction itself is liable to variation, according to the obliquity of the surfaces; for since the friction is nearly proportional to the mutual pressure, it will be greater or less, as the direction of these surfaces is more or less inclined to the radii, the force of rotation being supposed to be given: and, what is of still more immediate importance to the resolution of the difficulty in question, the direction of the force, by which the one wheel acts on the other, is not to be considered as perpendicular to the surface of the teeth, but as oblique to it, being so situated as to oppose the joint result of the direct resistance and the friction; that is, as being inclined to the surface in a certain constant angle, which a late anonymous writer has called the angle of repose, and which is equal to the inclination of a plane, on which one

of the substances concerned would begin to slide on the other by its gravitation.

“ Let the tooth A impel the tooth B with the given force A C, perpendicular to the common surface of the teeth; make CAD equal to the angle of repose, then the force must act in the direction A D, and making C D parallel



to the radius A E, A D will be the actual pressure: then drawing D F parallel to

the radius A G, A F will be the effective force in the direction A C, and F C will be the loss by friction. Again, if B impel A, the angle of repose must lie on the other side of A C, and C H must be parallel to A G, and H I to A E, and the friction in this case will be I C, which is obviously less than F C.

“Hence we may easily calculate the magnitude of the resistance F C, or I C, produced by friction, calling the force A C unity; for C D becomes $\frac{s. CAD}{s. ADC}$, and

$$F C = C D \cdot \frac{s. CDF}{s. CFD} = \frac{s. CAD}{s. ADC} \cdot \frac{s. CDF}{s. CFD} =$$

$$\frac{s. CAD}{s. (CAE + CAD)} \cdot \frac{s. GAE}{s. GAC}; \text{ and in the same}$$

$$\text{manner } I C = \frac{s. CAH}{s. AHC} \cdot \frac{s. CHI}{s. CIH} = \frac{s. CAD}{s. (GAC + CAD)} \cdot \frac{s. GAE}{s. EAC}.$$

Both these quantities vary ultimately as the angle formed by the radii, and vanish when the point of contact is in the line of the centres; and in this case the common theory agrees with this calculation. When C A E is always a

right angle, as in the epicycloidal tooth commonly recommended, the friction FC varies, in the different positions of the teeth, as $\frac{s. GAE}{s. GAC}$, or as $\cot. GAC$; that is, if AK be made constant, as KL .

“ Since therefore it is demonstrable, that the friction is always greater in approaching the line of the centres, than at an equal distance beyond it, it must obviously be desirable that the contact should be rather after than before the passage of the teeth over that line, although it is better that it should be at a small distance before, than at a much greater distance beyond it. Hence, the impelling teeth ought to be of such a form, as to accelerate the motion of the impelled a little before, and a little more after, the passage of the line of the centres, and then to retard it again, so that the next tooth may succeed to a similar operation.

“ A wheel acting on a trundle, with

cylindrical staves, has in this respect an advantage over two wheels with teeth, since the curve, fitted for impelling the trundle, is adapted only to act on it beyond the line of the centres. This curve may however be formed more easily, and at the same time more advantageously, than by the method which has hitherto been recommended: for if we employ an epicycloid described by the rolling of a circle, which would just touch the internal surface of all the staves of the trundle, on the circumference of the wheel, the trundle will at first be accelerated a very little, and will then be allowed to fall back from each tooth to the succeeding one, soon after its passage over the line of the centres. The same form will also answer very well, when the trundle is to impel the wheel, although this mode of action produces a greater friction than the former.

“ A similar advantage may be obtained in teeth of any other form, by finishing

them in such a manner as to project a very little beyond the regular outline, at the point which is intended to come into contact a little beyond the line of the centres. Such a corrected outline may be described at once, if it be required. If the tooth is to be formed into an involute of a circle, having fitted a thread or fine wire to the circumference of the wheel, find the point of contact at the instant when the end of the wire is describing the part of the tooth, which is to act at, or a little before, the line of the centres; cut off from the wheel, beyond this point, an arc equal to the distance of the centres of two adjoining teeth, and fix a pin in the tangent at the same point, that is, in the continuation of the part of the wire which is unrolled, at such a distance as just to stretch the part which is left loose by the removal of the arc: the pin thus fixed, and the remainder of the circle, will serve as bases for continuing the evolution of the wire, and the description of the tooth. The same position of

the wire will show the outline of a basis proper for describing, by means of a circle rolled on it, the curve which must be substituted for the form of any epicycloidal tooth, which might have been described by causing the same circle to roll on the simple circumference of the wheel as a basis; the curved part of the tooth beginning, in this case, at the point of contact first mentioned.

“ If it be objected, that in such an arrangement, the equability of the motion would be lost, and a shake would be created ; it may be answered, that the inequality would be utterly imperceptible in practice. But I do not know, that the form, thus determined, would have any material advantage over teeth made as short as possible, or so cut away as not to act before the passage of the line of centres, which may easily be done in all cases, nearly in the same way as you have shown with respect to epicycloidal teeth.

“ The advantage of dividing the pressure among several teeth ought not to be purchased at the expense of an increase of friction, since the property of greater durability may be obtained, in an equal degree, by simply making the wheels thicker, without materially adding to the friction: and in fact, although the momentary pressure on each tooth may be lessened by dividing it, yet its duration is increased in the same proportion.

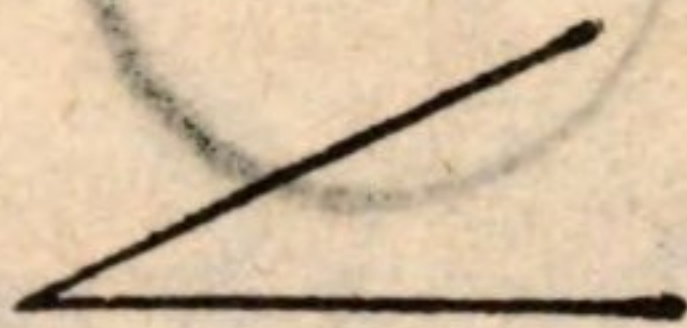
“ I must beg leave to observe, that the form proper for the teeth of a pinion, acting on a rack, is the involute of a circle, and not a cycloid. The cycloid would be a proper form for the teeth of the rack, if they were intended to impel the pinion.

“ It has been remarked that the form of the involute of a circle is not immediately deducible from the general principle of Lahire; and the remark is strictly true, since the curves, formed, according to that

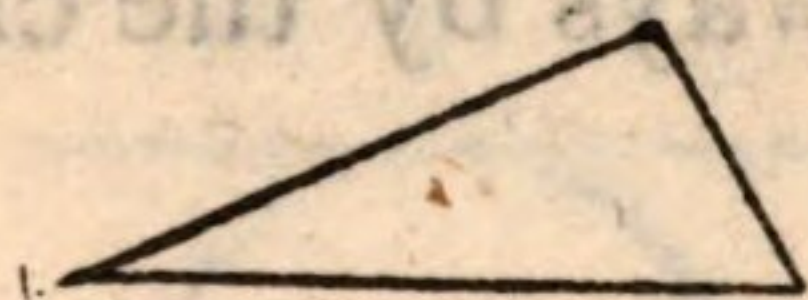
principle, from two contiguous circles as bases, could not act on each other without a further separation of the centres, which would render the demonstration inadequate. But I have observed in the Additions to my second volume, p. x. that the principle may be extended to any other curves, as well as circles and straight lines: and if we employ an equiangular spiral, instead of a straight line, we shall have the involutes, exactly as they are recommended for practice."

SUPPLEMENTARY DEFINITIONS.

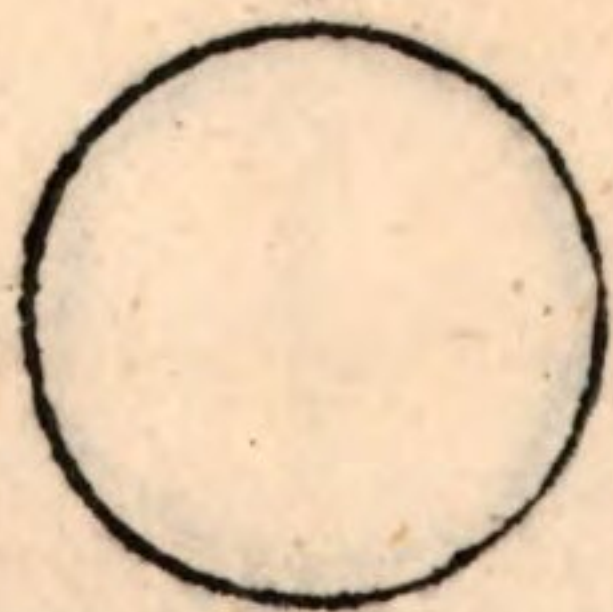
1st. An angle is the inclination of two lines to one another which meeting do not lie in one line.



2. A triangle is a figure contained by three straight lines.



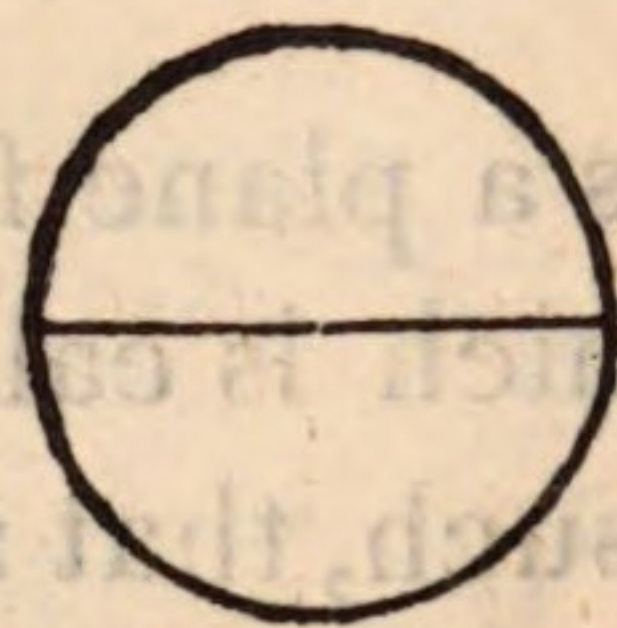
3. A circle is a plane figure contained by one line, which is called the circumference, and is such, that all straight lines drawn from a certain point within the figure to the circumference, are equal to one another, and this point is called the centre of the circle.



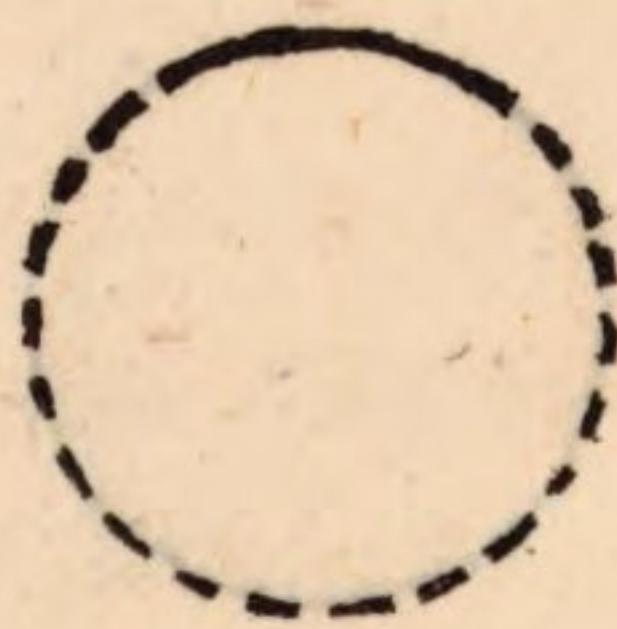
4. The radius of a circle, is a right line drawn from the centre to the circumference. The word, radii, is used, when more than one such line is spoken of.



5. The diameter of a circle, is a right line drawn through the centre, and terminated both ways by the circumference.



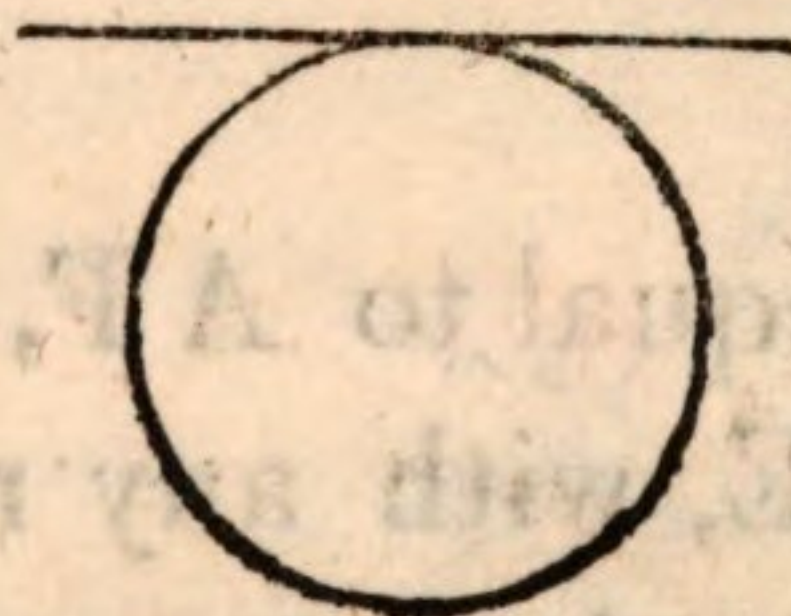
6. The arc of a circle is any part of its circumference.



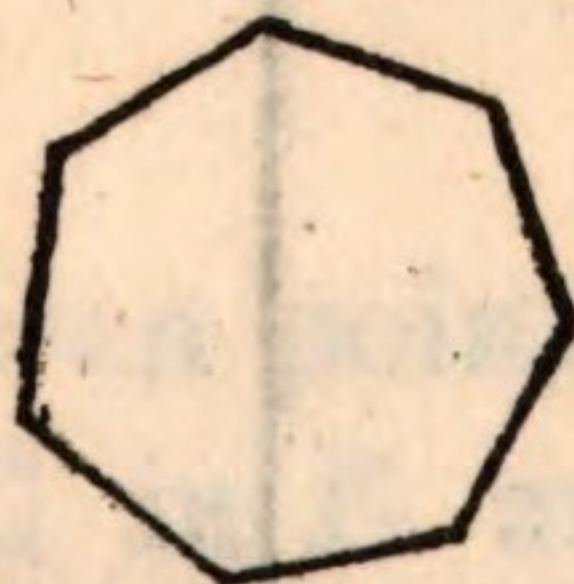
7. A chord of an arc, is a right line joining the two extremities of the arc.



8. A tangent of a circle is a right line, which passes through a point in the circumference without cutting it.

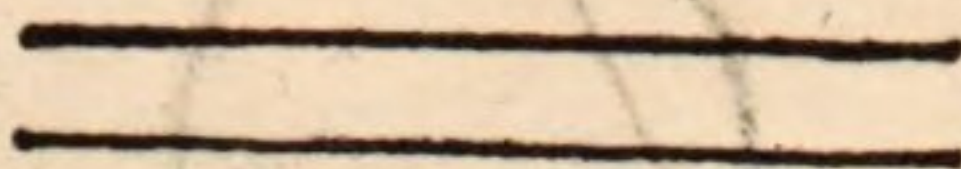


9. A polygon is a figure, having more than four sides. The term is seldom applied to figures that have less than five sides.



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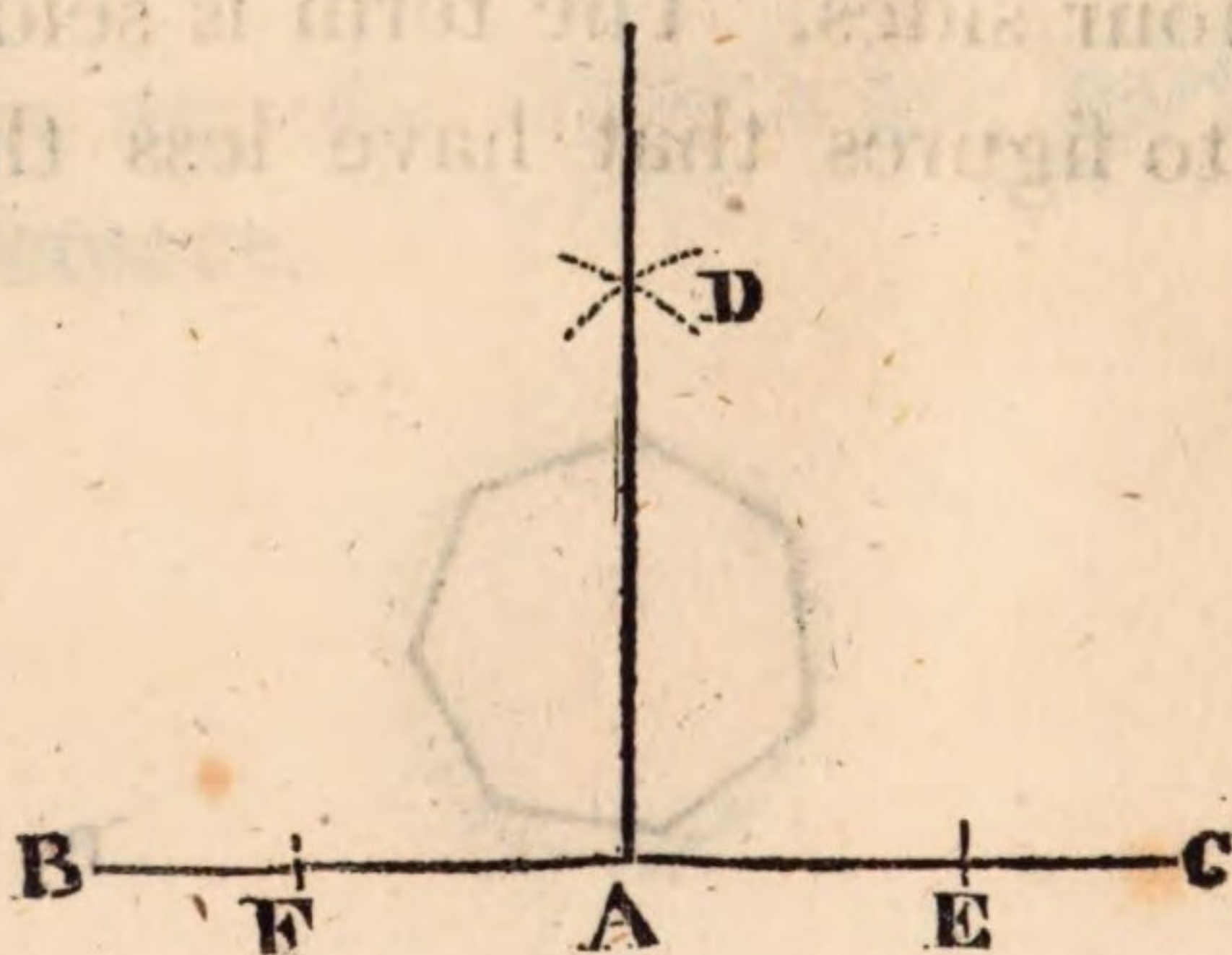
10. Parallel right lines are such as are in the same plane, and which, being continued ever so far either way, never meet.



11. The term perpendicular is the same with square, as used by workmen.

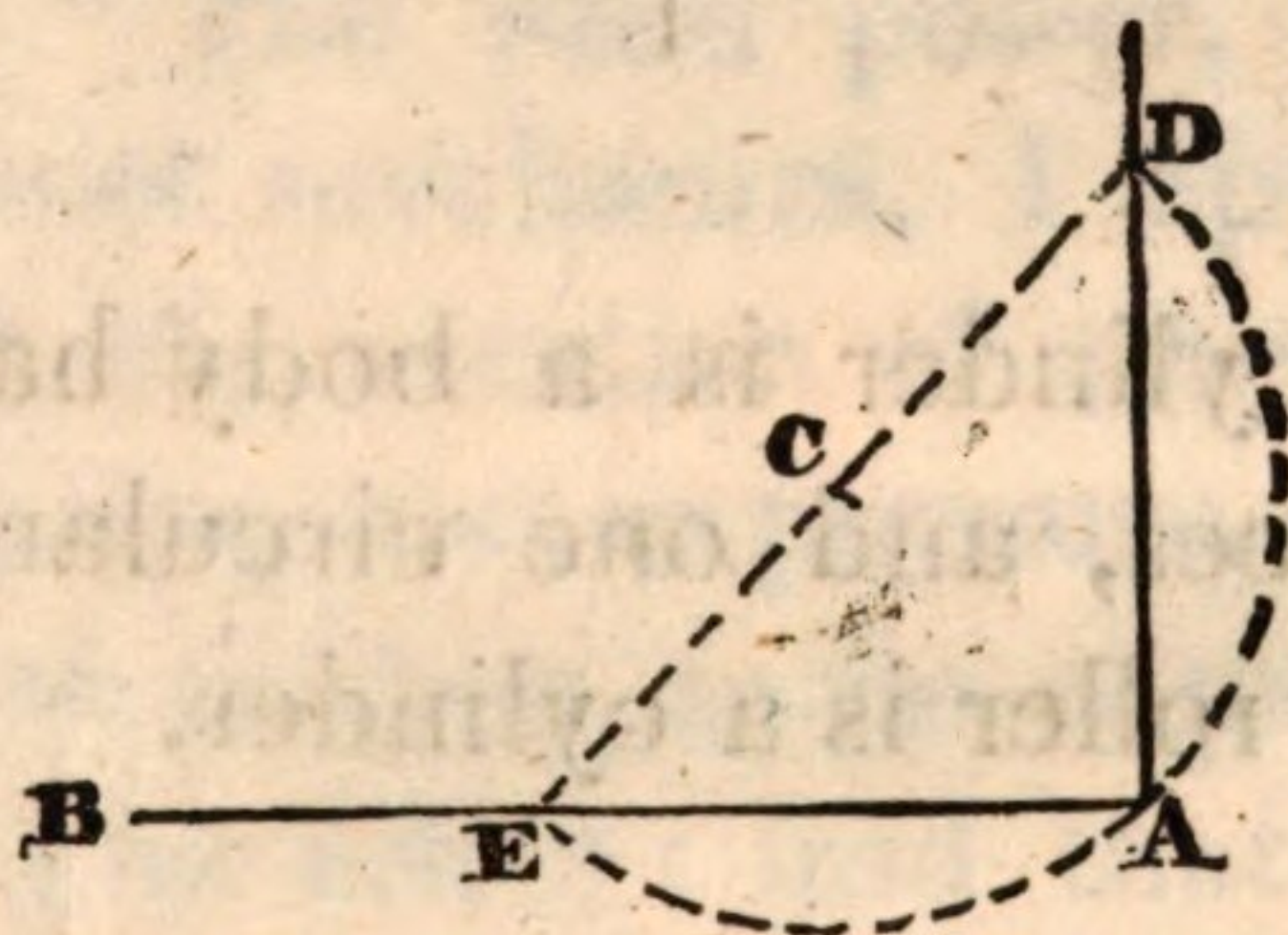
In order to draw from a given point, A, in a given line, B C, another line perpendicular to it,

Take A E, equal to A F, and from the points F and E, with any radius greater than A B, make the intersection D; draw D A, which will be perpendicular to B C.



If it be required, from the end of a given right line, $A B$, to raise a perpendicular,

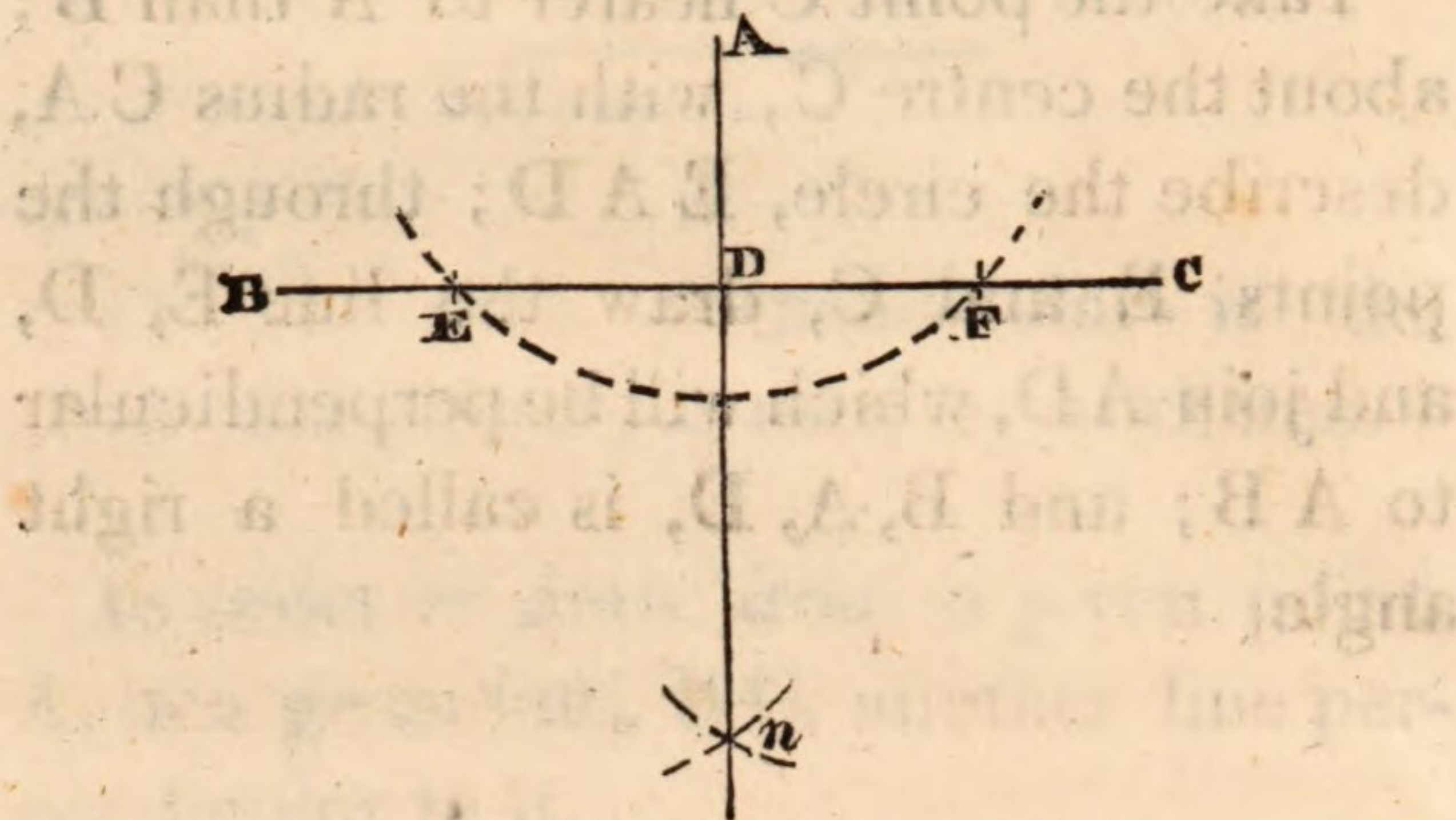
Take the point C nearer to A than B ; about the centre C , with the radius $C A$, describe the circle, $E A D$; through the points E and C , draw the line E, D , and join $A D$, which will be perpendicular to $A B$; and B, A, D , is called a right angle.



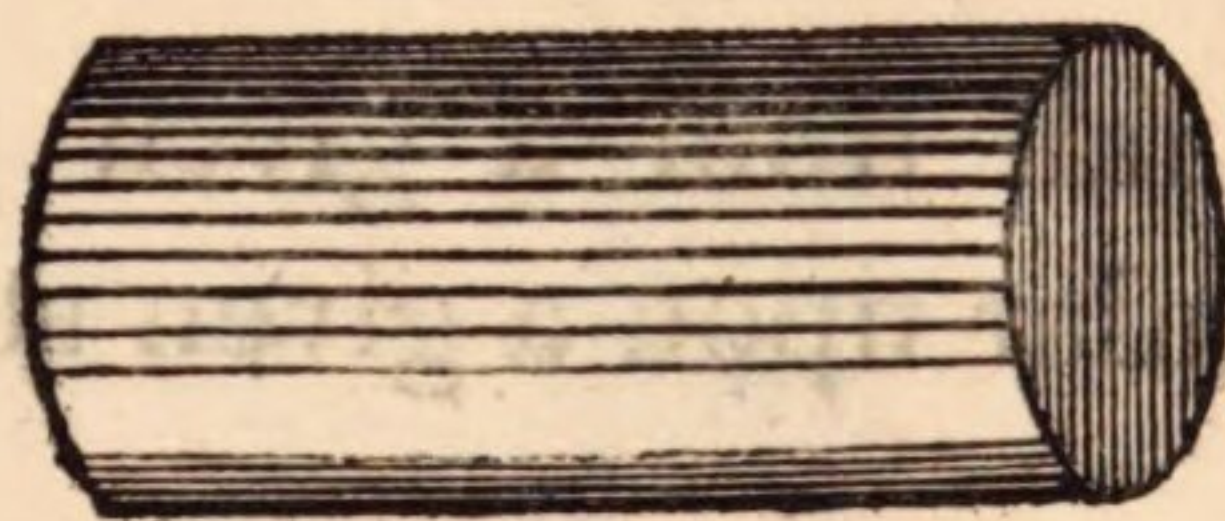
To let fall from a given point A , a perpendicular upon a given straight line, $B C$.

About the given point, describe a circle cutting $B C$ in E and F ; from the

points E and F, make the section n , and draw the line A D from A towards n , and A D is the perpendicular required.

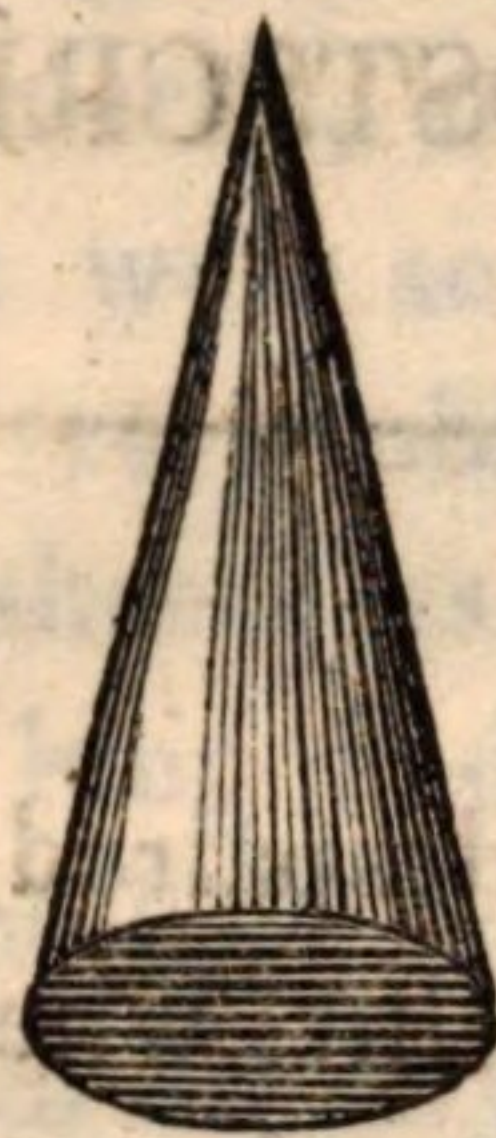


12. A cylinder is a body having two flat surfaces, and one circular. For instance, a roller is a cylinder.



13. A cone is a solid body, of which

the base is a circle, and which ends in a point.



14. Velocity is equivalent to speed.

15. Mr. Smeaton thus defines the term power: “The word power, as used in
“ practical mechanics, I apprehend to
“ signify the exertion of strength, gravi-
“ tation, impulse, or pressure, compound-
“ ed with motion, to be capable of pro-
“ ducing an effect ; and that no effect is
“ properly mechanical, but what requires
“ such a kind of power to produce it.”

16. A proposition is a sentence in which any thing is affirmed.

17. A corollary is an inference or deduction.

POSTSCRIPT.

I HAD almost despaired of seeing the preceding Essay in print; when after being about eighteen months in the press, I was at last informed that it was nearly all printed off, and that I might have the opportunity of making any additions that might occur. I do not think, that I can here express myself better than in the words of Professor Leslie, which he uses on a similar occasion.

“ My distance from the press, besides occasioning other inconveniences, had prevented me from bestowing the degree of correction which I was solicitous to attain. Some mistakes of a more important kind may have escaped notice, which I should probably have rectified, if the impression had been more immediately under my command.”*

I, however, have much satisfaction in ac-

* Leslie's Experiments on Heat, p. 9.

knowledging, that on this occasion, I have possessed one very great advantage. Dr. Young (of whose merits, as a mechanician and philosopher, it were superfluous for me to speak), was so very obliging as to examine most of the sheets as they came from the press. Having long had it in contemplation, to make a collection of facts respecting wheels actually in use in millwork, and by arranging them, to endeavour to draw some useful practical inferences, I take the present opportunity of offering to the public the hints on that subject, which may be found in the following Appendix.

I could have wished, that time and other circumstances had allowed me to have entered more minutely into the subject. Yet hoping, that these hints, even in their present state, may lead to a fuller investigation, I trust, they will not be altogether without some advantage. As, I believe, nothing exactly of the same kind, has hitherto been published, it is hoped, that these observations will be received by the public with indulgence and due allowance for their unavoidable imperfections.

With respect to the elementary propositions which we have to guide us in this inquiry into the proportional strength of the teeth of wheels, I shall not enter into their demonstrations. To the artisan, unacquainted with mathematics, they would be unintelligible; and the mathematician can either demonstrate them himself, or have recourse to those elementary writings where the demonstrations may be found: of these last, as being more generally accessible, I have referred to "*Emerson's Mechanics*," quarto edition.

Glasgow,
5th April, 1808.

APPENDIX.

A PRACTICAL INQUIRY RESPECTING THE STRENGTH AND DURABILITY OF THE TEETH OF WHEELS USED IN MILLWORK.

HAVING treated of the forms of the teeth of wheels, I come now to consider their proportional strength with relation to the resistance they have to overcome.

I am aware, that owing to a great variety of circumstances, this subject is involved in much difficulty, and that it is no easy task to form any general rule with regard to the pitches and breadths of the teeth of wheels. I do not pretend to more than a mere approximation towards general rules; yet, were this judiciously done, I am of opinion, that it might be useful to the millwright, who has not had leisure or opportunity for scientific inquiries. A rule, though not absolutely per-

fect, is better in all cases, than to have no guide whatever.

And it is too evident to require proof, that it is essential to the beauty and utility of any machine, that the strength and bulk of its several parts, be duly proportioned to the stress or wear to which the parts may be subject.

Some general observations on the wheel work of mills, will serve greatly to simplify our inquiries on the subject.

General Observations on the Wheel Work of Mills.

Mistaken attempts at economy have often prompted the use of wheels of too small diameter. This is an evil which ought carefully to be avoided. Knowing the pressure on the teeth, we cannot with propriety reduce the diameter of a wheel below a certain measure.

Suppose, for instance, a water wheel of 20 horses' power, moving at the pitch line with a velocity of $3\frac{1}{2}$ ft. pr. second. It is known, that a pinion of 4 feet diameter, might work

into it, without impropriety; but we also know, that it would be exceedingly improper to substitute a pinion of only one foot diameter, although the pressure and velocity at the pitch lines in both cases would be, in a certain sense, the same. In the case of the small pinion, however, a much greater stress would be thrown on the *journeys*, (or *journals*) of the shaft. Not, indeed, on account of torsion or twist, but on account of transverse strain, arising, as well from greater direct pressure, as from the tendency which the oblique action of the teeth, particularly when somewhat worn, would have to produce great friction, and to force the pinion from the wheel, and make it bear harder on the journals. The small pinion is also evidently liable to wear much faster, on account of the more frequent recurrence of the friction of each particular tooth.

That these observations are not without foundation, is known to millwrights of experience. They have found a great saving of power, by altering corn mills, for example, from the old plan of using only one wheel and pinion, (or *trundle*,) to the method of bringing up the motion, by means of more

wheels and pinions, and of larger diameters and finer pitches.

The increase of power has often by these means been nearly doubled, while the tear and wear has been much lessened; although it is evident, the machinery, thus altered, was more complex.

The due consideration of the proper communication of the original power, is of great importance for the construction of mills on the best principles. It may easily be seen, that in many cases, a very great portion of the original power is expended, before it is actually applied to the work intended to be performed.

Notwithstanding the modern improvements in this department, there is still much to be done. In the usual modes of constructing mills, due attention is seldom given to scientific principles. It is certain, however, that were these principles better attended to, much power, that is unnecessarily expended, would be saved. In general, this might be in a great measure obtained, by bringing on the desired motions in a gradual manner, beginning with the first very slow, and gradually

bringing up the desired motions, by wheels and pinions of larger diameters. This is a subject which should be well considered before we can determine, in any particular case, what ought to be the pitch of the wheels. In the case above alluded to, where the supposition is a pinion of 4 feet diameter, or of 1 foot diameter; it is obvious, that the same pitch for both would not be prudent. That for the small pinion, ought to be much less than that which might be allowed in the case of the larger pinion. It is also equally obvious, that the breadth of the teeth, in the case of the small pinion, ought to be much greater than that in the case of the larger pinion.

It is evident, however, that although great advantage may often be derived from a fine pitch, that there is a limit in this respect, as also with regard to the breadth. We shall endeavour to find some trace of this limit in what follows; and that we may the better do this, we shall call in the aid of propositions, which are true with respect to pieces of timber, or metal, subjected to ordinary cases of pressure. It is allowed, that they cannot here, in strictness, be *demonstrated*, as appli-

cable to wheelwork. Yet they will, for want of better light, serve at least to prevent any material practical error with regard to the strength of the teeth of wheels. For it is to be remembered, that we are not so much here in search of truths of curious or profound mathematical speculation, as of that kind of evidence, of which the subject admits, and which may be sufficiently satisfactory for any practical purpose.

I would have it, however, understood, that we suppose the diameters made sufficiently great to prevent the evils which we have already noticed, and in the annexed table, are some examples in actual use, which have been found in practice sufficiently durable. I would particularly recommend attention to those of Messrs. Boulton and Watt, whose most extensive practice, as well as scientific knowledge, renders their work a model well worthy the attention of millwrights.

As cast iron pinions are now generally used, and as the teeth of the pinion are most subject to wear, I think we are safe in the present inquiry, in considering them all as cast iron.

The laws to which I have alluded in this investigation are these :

PROPOSITION I.

*“ The Strength of any Piece of Timber, or Metal, whose Section is a Rectangle, is in direct Proportion to the Breadth, and as the Square of the Depth.” **

Hence may be inferred, that the strength of teeth of wheels, moving at the same velocity, and under the same circumstances, is directly in proportion to their breadth, and as the square of their thickness. Thus, for example, if we double the breadth, we only double the strength ; but if we double the thickness, in other words, double the pitch, keeping the original breadth, we increase the strength four times.

For although when wheels are working accurately, the strain is at the same time divided over several teeth, yet as a very small inaccuracy, or even the interposition of any

* See Emerson, p. 95.

small body, such as a chip of wood or stone, throws the whole stress upon a single tooth in practice; therefore, and in order to simplify this case, we may consider the strength of a single tooth, as resisting the pressure of the whole work.

But as the length of teeth commonly varies with the pitch, this circumstance must be taken into account, and the most simple view we can take of it seems to be, that of having the strain of each tooth thrown all to the outward extremity, we have then the following proposition to guide this part of our inquiry.

PROPOSITION II.

*If any Force be applied laterally to a Lever or Beam, the Stress upon any Place, is directly as the Force and its Distance from that Place.**

* See Emerson, p. 96.

PROPOSITION III.

*The Pitch being the same, the Strength is
inversely as the Velocity.**

For example—if the pitch lines of one pair of wheels be moving at the rate of 6 feet in a second, and another pair of wheels, in every other respect under the same circumstances, be moving at the rate of 3 feet in a second, the stress on the latter, is double of that on the former.

We shall confine our attention for the present, to wheels having cast iron teeth ; and, in order to take experience as our guide, several examples in the annexed tables, actually in use, are selected.

The pitch, velocity, and strain, are all stated ; the strain is measured by the *horses' power*, at which the resistance is valued. Horse's power is a term now in general use, to express the force required, in order to drive

* See Emerton, p. 179.

any kind of mill, and it may be proper here to give some further account of it.

Horse's Power.

Although horses are not all of one strength, yet there is a certain force now generally agreed upon among those who construct steam engines, which force is denominated a *horse's power*, and hence, steam engines are distinguished, in size, by the number of horses' power to which they are said to be equal.

The measure of a mechanical effect equal to a horse's power, has been much disputed: this I believe to be a matter of little consequence, if the measure be generally understood, since there is no such thing as bringing this into any real measure. Some horses will work double of others, and horses in one country, will work more than those of another. Desaguliers's measure is, that a horse will walk at the rate of $2\frac{1}{2}$ miles per hour, against a resistance of 200lbs, and which gives as a number of comparisons 44000, that is, the raising of 1lb 44000 feet in a mi-

nute; or what amounts to the same, the raising of 44000lbs 1 foot in a minute.

Emerson's measure is the same as Desaguliers's, (see Emerson's Mechanics, p. 178), and Mr. Smeaton's result is 22916lbs under the same circumstances.

Mr. Watt has found, from repeated experiments, that 33000lbs, 1 foot per minute, was the average value of a horse's power; but, I believe, that his engines were calculated to work equal to 44000lbs 1 foot per minute.

But, that he allows only 33000 in his calculations, applied to mills, considering the difference as being lost in the friction of the engine itself.

It is common in practice, to reckon, that it requires one horse's power, to drive 100 spindles with preparation of cotton water twist.

1000 spindles with preparation cotton mule yarr.

75 spindles with preparation flax yarn.

I beg leave here to make the following extract, on the subject of animal force, from Dr. Young's Natural Philosophy, Vol. II. p. 165.

“ In order to compare the different estimates of the force of moving powers, it will be convenient to take a unit which may be considered as the mean effect of the labour of an active man, working to the greatest possible advantage, and without impediment; this will be found, upon a moderate estimation, sufficient to raise 10 pounds 10 feet in a second for 10 hours in a day; or to raise 100 pounds, which is the weight of 12 wine gallons of water, 1 foot in a second, or 36000 feet in a day, or 3 600 000 pounds, or 432000 gallons, 1 foot in a day; this we may call a force of 1, continued 36000.”

“ Immediate force of men, without deduction for friction.

	Force.	Continuation	Day's Work.
A man of ordinary strength, can turn a winch, with a force of 30 pounds, and with a velocity of $3\frac{1}{2}$ feet in 1" for 10 hours a day—Desaguliers.	1.05	10h	1.05
Two men working at a windlass, with handles at right angles, can raise 70 pounds more easily than one can raise 30—Desaguliers .	1.22		1.22
For a short time, a man may exert a force of 80 pounds, with a fly, when the motion is pretty quick—Desaguliers	3.	1"	

Performance of Men by Machines.

	Force.	Continuance	Day's Work.
A man can raise, by a good common pump, a hogshead of water 10 feet high in a minute, for a whole day—			
Desaguliers	.875		.875
A horse can draw, with a force of 200 pounds, $2\frac{1}{2}$ miles an hour for 8 hours in the day			
	7.33	8h.	5.87
With a force of 240, only 6 hours—Desaguliers			
	8.8	6h.	5.28
By means of pumps, a horse can raise 250 hogsheads of water, 10 feet high, in an hour—Smeaton's Reports .			
	3.64	1h.	

*Explanation of the Table of Wheels in
actual Use in Millwork.*

The wheels are all reduced to what may be called one denomination.

First—By proportioning their breadths all to what they should be to have the same strength, if the resistance were equal to the work of a steam engine of ten horses' power.

Secondly—By supposing their pitch lines all brought to the same velocity of 3 feet per second, and proportioning their breadths accordingly. I have chosen this particular velocity of 3 feet per second, because it is the velocity very common for overshot water wheels.

Such cases as appear to have worn too rapidly, are marked, which may tend to discover the limit in point of breadth.

136 *An Essay on the Teeth of Wheels.*

Column	1	Contains the Horse's power.
_____	2	_____ Pitch in inches.
_____	3	_____ { Breadth of teeth in inches.
_____	4	_____ { Number of teeth of wheel.
_____	5	_____ { Revolutions of wheel per minute.
_____	6	_____ Diameter of wheel.
_____	7	_____ { Number of teeth of pinion.
_____	8	_____ { Revolution of pi- nion per minute.
_____	9	_____ Diameter of pinion.
_____	10	_____ { Breadth propor- tionate to 10 horses' power, and at the pre- sent velocity.
_____	11	_____ { Present velocity per second in feet.
_____	12	_____ Breadth in Inches

proportionate to 10 horses' power, at 3 feet per second. That is, all the cases reduced to the same denomination.

*Observations on the Table of Wheels in
actual Use in Millwork.*

Having reduced the examples in the table, in the manner already described, to one denomination, the results approach nearer, considering all circumstances, than could have been expected.

1st, In two of the cases, viz. B and F, it appears, that the wheels were rather too narrow for their work. These, however, have been working about sixteen years, and may yet continue for a long time.—B is 4.489 inches in breadth, when reduced to 10 horses' power, at 3 feet per second.—D is 7.27 inches in breadth. The pitch of both is three inches.

Three inches being a pitch in very general use for the first motion of mills, could the proper breadth for this pitch be ascertained, it would serve as a very useful standard.

Rules to be of practical use, must be easy of remembrance, as well as easy of application.

Let us, therefore, assume a simple standard, and try how far it will bear the test of experience; for, as Du Buat justly observes, "It is an excellent method, in the research of obscure difficult truths, to suppose a theory pre-existent, founded upon the most probable principles, from which may be determined, the choice of some direct experiments, proper to evince the fallacy or accuracy of the principles proposed."

The actual cases in the table, may be considered as satisfactory experiments. Let us, therefore, try the following simple rule, and compare some of its results with those cases.

Rule for pitch of three inches, when the velocity is three feet per second, at the pitch line.

Make the teeth as many inches broad as the number of horses' power which it has to resist.

For example, for nine horses' power, make the teeth nine inches broad.

Taking this example then as a point from

which to set off, and supposing the same breadth of *nine inches* constant, we shall, in the first following table, state various pitches, and (by proposition I.) shall first square these pitches, to find the number of horses' power, equal to the strength and durability of the pitch, when the teeth are all of one length. The strength thus found, will be inserted in column Y.

But, as the lengths generally vary as the pitches, taking the same point, (three inches pitch, nine inches broad) from which to set off, we shall diminish the value of the strength, as we ascend, and increase as we descend, agreeably to Proposition II. The results will be found in column Z.

But as in this investigation durability is of equal importance with strength, perhaps the true proportion may be somewhere between the results in the columns Y and Z.

Description of the Table of Pitches.

In all the following tables, the column W contains the pitch in inches.

Column X contains the breadth of the teeth also in inches.

Column Y is formed upon the supposition, that the teeth of all the pitches were of the same length, and contains the strength and durability of the teeth valued in horses' power.

Column Z contains the strength and durability of the teeth, also valued in horses' power, upon the supposition, that the lengths of the teeth are in the same proportion to one another, as the pitches; and that the strength (by Prop. II.) is inversely as the length. Having taken a three inch pitch as our standard, the two last columns, Y and Z, exactly coincide for that pitch; the column, Z, decreasing upwards from that point, and increasing downwards in an inverse ratio of the lengths.

It is evident, that the tables upon these principles already laid down, might be greatly extended. But it is hoped, that these will be sufficient for our present purpose.

I. *Table of Pitches.*

The velocity of the pitch line, being three feet per second, and the breadth of the teeth nine inches.

	W	Y	Z
Pitch in Inches.	Value of strength in horses' power.	Value of strength in horses' power.	Value of strength in horses' power.
4	16.	12.	12.
$3\frac{1}{2}$	12.25	10.5	10.5
3	9.	9.	9.
$2\frac{1}{2}$	6.25	7.5	7.5
2	4.	6.	6.
$1\frac{1}{2}$	2.25	4.5	4.5
1	1	3.	3.

Supposing again, that the pitches were the same as in the first table, and that the breadths were made, in each particular case, just double the pitch, then the horses' power would vary as in the following table, which is calculated by taking the above table, and

by direct proportion, finding the horses equal to each particular breadth, when the breadth is just double the pitch.

II. *Table of Pitches.*

The velocity being three feet per second, and the breadth of the teeth double each pitch.

W X Y Z			
Pitch in inches.	Twice the pitch in breadth.	Value of strength in horses' power.	Value of strength in horses' power.
4	8	14.22	10.66
$3\frac{1}{2}$	7	9.53	8.17
3	6	6.	6.
$2\frac{1}{2}$	5	3.47	4.16
2	4	1.77	2.65
$1\frac{1}{2}$	3	.75	1.5
1	2	.22	.66

The strength being directly as the breadth, (by Prop. I.) it is easy from this to find, by the Rule of Three direct, the horses' power equal to any given breadth of these pitches.

The strength being inversely as the velocity, (by Prop. II.) the horses' power equal to any of these pitches, and breadth, may be easily found for any other velocity.

But, perhaps, it will be of advantage to take a different view of the subject, by fixing upon a case in the table of wheels in actual use, and proportioning breadths and pitches from it for various velocities, resistances, &c.

For this purpose we shall select the case H, erected by Messrs. Boulton and Watt, being a pitch of three inches, the teeth 8 inches broad, moving with a velocity of 11 feet per second, and having a resistance valued at the power of 46 horses.

Then by following a similar procedure, with the two foregoing tables, we have the tables III. and IV. proportionate to the case H, at a velocity of 11 feet per second.

III. *Table of Pitches*

Proportionate to H in the Table of Wheels.
The breadth of teeth (8 inches) and velocity
(eleven feet per second) being constant.

W Y Z		
Pitch in inches.	Horses' power.	Value of strength in horses' power.
4	81.77	61.33
$3\frac{1}{2}$	62.61	46.95
3	46.	34.5
$2\frac{1}{2}$	31.94	23.54
2	20.44	15.33
$1\frac{1}{2}$	11.5	8.62
1	5.11	3.84

IV. *Table of Pitches,*

Proportionate to H, the breadth being in this case double the pitch. The velocity being constant (eleven feet per second), but the breadths double.

W X Y Z

Pitch in inches.	Breadth double the pitch in inches.	Horses' power.	Value of strength in Horses' power.
4	8	81.77	61.33
$3\frac{1}{2}$	7	54.78	46.95
3	6	34.5	34.5
$2\frac{1}{2}$	5	19.95	23.54
2	4	22.	15.33
$1\frac{1}{2}$	3	4.31	8.62
1	2	1.28	3.84

But it may be satisfactory, in order to compare with the tables, first and second, to reduce the velocity to three feet per second.

The two following tables, therefore, are calculated accordingly at that velocity.

V. Table of Pitches,

Proportionate to H, at a velocity of three feet per second ; the breadth being constantly eight inches.

W Y Z		
Pitch in inches.	Horses' power.	Value of strength in horses' power.
4	22.30	16.72
$3\frac{1}{2}$	17.07	12.79
3	12.54	9.40
$2\frac{1}{2}$	8.71	6.53
2	5.57	4.17
$1\frac{1}{2}$	3.13	2.34
1	1.39	1.02

VI. Table of Pitches,

Proportionate to H, at a velocity of three feet; the breadths being double each pitch.

W X Y Z

Pitch in inches.	Breadth of teeth in inches.	Horses' power.	Value of strength in horses' power.
4	8	22.30	16.72
$3\frac{1}{2}$	7	14.93	12.79
3	6	9.4	9.4
$2\frac{1}{2}$	5	3.44	6.53
2	4	2.78	4.17
$1\frac{1}{2}$	3	1.17	2.34
1	2	0.34	1.02

From a comparison of the second table with the sixth, it appears, that the rule we have annexed, is at least safe in point of strength; for by that rule, a *three inch pitch, six inches broad, is equal to a strain of six horses*. Whereas, in the sixth table, the same pitch and velocity, at six inches

breadth, has strength valued at nine and nearly a half horses' power.

But when we consider how much more liable, from sand, &c. teeth attached to water wheels are to wear, than those which are properly greased and free from sand, the results correspond as nearly as could be expected.

So that taking H as a standard, we may conclude, that, *for a pitch of three inches*, with a velocity of three feet per second, every inch of breadth may be valued at one and a half horses' power.

The first rule I think therefore may safely be followed for teeth attached to water wheels, and the above conclusion for wheels in all situations where they are properly greased and free from sand.

The conclusions here drawn, will, I think, give teeth sufficiently durable and strong for the work which they may have to perform. This first conclusion gives, perhaps, too great a result: how much may with prudence be deducted from the results of ei-

ther, those of experience will determine. But to the young millwright, I would advise, of the two extremes, rather to err in making his work too strong. Durability ought not for a moment to be out of sight in the arrangements of wheel work; there are many parts of machines subjected to great stress, but not liable to wear; whereas the teeth of wheels, the moment they begin to act, begin to change from their original form, and to become progressively less strong.

In millwork, at present, the breadth of the teeth, as commonly executed by the best masters, seems to be from about twice to thrice the pitch.

It is, perhaps, not easy to determine what proportion is on the whole the most advantageous for the breadth of teeth. A fine pitch, on the one hand, gives a smooth motion, and the teeth will rub less on each other; but an increase of breadth increases in some degree the friction.* The durability, as well as the strength of teeth, is perhaps nearly in direct proportion to their breadth.

* See p. 105.

After sending the foregoing "*Inquiry respecting the Strength of the Teeth of Wheels*" to the press, Mr. John Roberton, engineer, perused a manuscript copy of it, and was so obliging as to communicate to me the substance of what follows. His rule, it will be readily perceived, is founded on the principles laid down in the "*Inquiry*;" but it is more simple, and perhaps more accurate than the mode of approximation which occurred to me. It is however satisfactory to find, that the table formed on his rule, (which from his experience he is of opinion cannot be far from the truth), very nearly coincides in its results with tables fifth and sixth. It may be observed, that he founds his calculations upon *the thickness of the teeth*, which in all cases he supposes a little less than half the pitch, which proportion is very common in practice. When both wheel and pinion, however, are of cast iron, it is evident, that being more liable to wear, the teeth of the pinion ought to be thicker than those of the wheel.

Construction of the following Table.

The thickness of the teeth in each of the lines, is varied one tenth of an inch. *The*

breadth of the teeth is always four times as much as their thickness. *The strength* of the teeth is ascertained by multiplying the square of their thickness into their breadth, taken in inches and tenths, &c. The *pitch* is found by multiplying the thickness of the teeth by 2.1. The number that represents the strength of the teeth, will also represent the number of horses' power, at a velocity of about four feet per second. Thus in the table where the *pitch* is 3.15 inches, the thickness of the teeth 1.5 inches, and the *breadth* 6. inches, the strength is valued at $13\frac{1}{2}$ horses' power, with a velocity of four feet per second at the pitch line,

1	2.1	4.41	8.82	13.23	17.64	22.05
2	4.2	17.64	35.28	52.92	70.56	88.20
3	6.3	39.69	79.38	119.07	158.76	198.45
4	8.4	70.56	141.12	211.68	282.24	352.80
5	10.5	110.25	220.50	330.75	440.10	550.35
6	12.6	158.76	317.52	476.28	635.04	794.40
7	14.7	216.09	432.18	648.27	864.36	1080.45
8	16.8	282.24	564.48	852.72	1136.64	1420.80
9	18.9	357.21	714.42	1071.63	1428.84	1785.75
10	21.0	441.00	882.00	1323.00	1764.00	2205.00
11	23.1	534.21	1068.42	1592.13	2119.44	2674.35
12	25.2	635.04	1276.32	1885.68	2494.56	3129.60
13	27.3	744.09	1495.38	2204.07	2898.24	3670.35
14	29.4	861.36	1726.72	2547.66	3331.20	4197.60
15	31.5	987.81	1970.61	2917.41	3794.40	4711.35
16	33.6	1123.44	2228.16	3313.92	4287.84	5211.60
17	35.7	1268.25	2500.02	3737.27	4811.52	5708.35
18	37.8	1422.84	2787.60	4188.48	5365.44	6201.60
19	39.9	1587.21	3091.32	4667.67	5949.60	6691.35
20	42.0	1760.40	3411.36	5175.00	6564.00	7278.00
21	44.1	1942.41	3747.84	5711.41	7208.64	7861.35
22	46.2	2133.24	4101.12	6277.86	7883.52	8441.60
23	48.3	2332.89	4471.68	6874.37	8588.64	9018.35
24	50.4	2541.36	4860.00	7500.96	9324.00	9601.60
25	52.5	2758.65	5266.50	8157.63	10090.56	10191.35
26	54.6	2984.76	5691.36	8845.32	10888.32	10788.00
27	56.7	3219.69	6134.88	9565.07	11717.28	11391.35
28	58.8	3463.44	6597.36	10317.96	12578.56	11991.60
29	60.9	3716.01	7079.22	11104.05	13472.16	12598.35
30	63.0	3977.40	7580.80	11924.40	14400.00	13211.60
31	65.1	4247.61	8092.32	12779.17	15363.36	13831.35
32	67.2	4526.64	8614.08	13668.42	16363.20	14457.60
33	69.3	4814.49	9146.40	14593.17	17400.48	15090.35
34	71.4	5111.16	9689.76	15553.44	18475.20	15729.60
35	73.5	5416.75	10244.58	16549.25	19587.36	16375.35
36	75.6	5731.36	10811.36	17580.66	20737.92	17027.60
37	77.7	6055.05	11390.52	18647.71	21926.88	17686.35
38	79.8	6387.84	11982.48	19750.44	23154.24	18351.60
39	81.9	6729.69	12587.76	20888.89	24420.00	19023.35
40	84.0	7080.40	13206.72	22063.00	25724.16	19701.60
41	86.1	7440.01	13839.60	23273.81	27066.72	20386.35
42	88.2	7808.44	14486.88	24521.28	28447.68	21077.60
43	90.3	8185.65	15148.08	25805.55	29867.04	21775.35
44	92.4	8571.60	15823.68	27126.64	31324.80	22479.60
45	94.5	8966.25	16514.16	28484.55	32820.96	23190.35
46	96.6	9369.60	17219.04	29879.32	34355.52	23907.60
47	98.7	9781.65	17938.80	31311.00	35928.48	24631.35
48	100.8	10202.40	18673.92	32780.64	37539.84	25361.60
49	102.9	10631.85	19424.88	34288.29	39189.60	26098.35
50	105.0	11070.00	20191.20	35834.00	40877.76	26841.60

A Table of Pitches of Wheels,

With the *breadth* and *thickness* of the teeth, and the corresponding number of horses' power, moving at the pitch line at the rate of three feet, of four feet, of six feet, and of eight feet per second.

Pitch in inches.	Thickness of teeth in inches.	Breadth of teeth in inches.	Strength of teeth, or number of horses' power at 4 feet per second.	Horses' power at 3 feet per second.	Horses' power at 6 feet per second.	Horses' power at 8 feet per second.
3.99	1.9	7.6	27.43	20.57	41.14	54.85
3.78	1.8	7.2	23.32	17.49	34.98	46.64
3.57	1.7	6.8	19.65	14.73	29.46	39.28
3.36	1.6	6.4	16.38	12.28	24.56	32.74
3.15	1.5	6.	13.5	10.12	20.24	26.98
2.94	1.4	5.6	10.97	8.22	16.44	21.92
2.73	1.3	5.2	8.78	6.58	13.16	17.34
2.52	1.2	4.8	6.91	5.18	10.36	13.81
2.31	1.1	4.4	5.32	3.99	7.98	10.64
2.1	1.0	4.	4.0	3.0	6.0	8.0
1.89	.9	3.6	2.91	2.18	4.36	5.81
1.68	.8	3.2	2.04	1.53	3.06	3.08
1.47	.7	2.8	1.37	1.027	2.04	2.72
1.26	.6	2.4	.86	.64	1.38	1.84
1.05	.5	2.	.5	.375	.75	1.

PRACTICAL OBSERVATIONS WITH REGARD
TO MAKING OF PATTERNS OF CAST IRON
WHEELS.

HAVING determined the pitch of the wheel strong enough for the purpose to which it is to be applied, the thickness of the tooth serves to regulate the proportionate strength of the other parts.

A very respectable millwright informs me, that he has for a considerable time adopted the following rule for determining the length of the teeth of wheels, the practical efficacy of which he has found quite satisfactory.

Rule—*Make the length of the teeth equal to the pitch, deducting freedom,* (by the freedom is meant the distance at the top of one tooth, and the root of another measured at the line of centres,) in other words, the distance from root to root of the teeth, at the line of teeth when the wheels are in action, exactly equal to the pitch.

For example—he makes the teeth of two

inches pitch, 1 inch and $\frac{1}{6}$ in length, which is allowing $\frac{3}{8}$ of freedom.

Another respectable millwright, who has had much experience, particularly in mills moved by horses, has for a considerable time past, made the teeth of his wheels in length only one half of the pitch, and works them as deep as possible, without the point touching the bottoms. Before he fell on this expedient, he found the teeth exceedingly liable to be broken from any sudden motion of the horses.

Indeed, upon reflection, it will be found there is no occasion for more freedom, than that the point of the tooth of the one wheel, shall just clear the ring of the other; more than this must only serve to weaken the teeth. The mode of geering, however, above alluded to, is more necessary in horse mills than where the moving power is steady and regular.

Hutton (on Clock-work,) recommends making the distance of the pitch line $\frac{1}{4}$ of what we call the thickness of the tooth. Thus, suppose the rule applied to a two inch

pitch and that the tooth and space were exactly equal, then the tooth would project $\frac{3}{4}$ of an inch beyond the pitch line, and its root would be as far within the pitch line, as to receive freely the tooth intended to act on it: suppose it also $\frac{3}{4}$, then the tooth would be $1\frac{1}{2}$ inch long, besides the freedom, which, as above, were $\frac{3}{16}$, the tooth would be in all $1\frac{1}{8}$ inch long.

But it is to be remarked, that the millwright, in making his pattern for a cast iron wheel, has to attend to a circumstance arising from the nature of that material. The pattern must not only be of such a form as to be sufficiently strong, calculating by the bulk of the parts, but also proportioned, so that when the fluid metal is poured in the mould, it may cool in every part nearly at the same time.

When due attention is not paid to this circumstance, as the metal is cooling, if it contract faster in one part than in another, it will be apt to break somewhere, just as a drinking glass is broken by suddenly cooling or heating in any particular part of it. In all patterns for cast iron, about $\frac{1}{4}$ of an inch to

the foot, should be allowed for the contraction of the metal in cooling.

Attention must also be paid to taper the several parts, so that they may rise freely without injuring the mould, when the founder is drawing them out of the sand. A little observation of the operations of a common foundry, will better instruct on this part of the subject than many words. We may observe, however, that about $\frac{1}{8}$ of an inch, in a depth of 6 inches, is commonly a sufficient taper.

Attending to those circumstances, I beg leave to offer the following proportions as having been found to answer in practice.

Make the thickness of the ring *a e* equal to the thickness of the tooth *a c* near its root. When the ring is made thinner than the root of the tooth, the ring commonly gives way to a strain, which would not break the tooth.

Make the arm, at the part where it proceeds from the ring, of the same breadth and thickness as the ring; and at the junction *e f*,

Fig. 1.

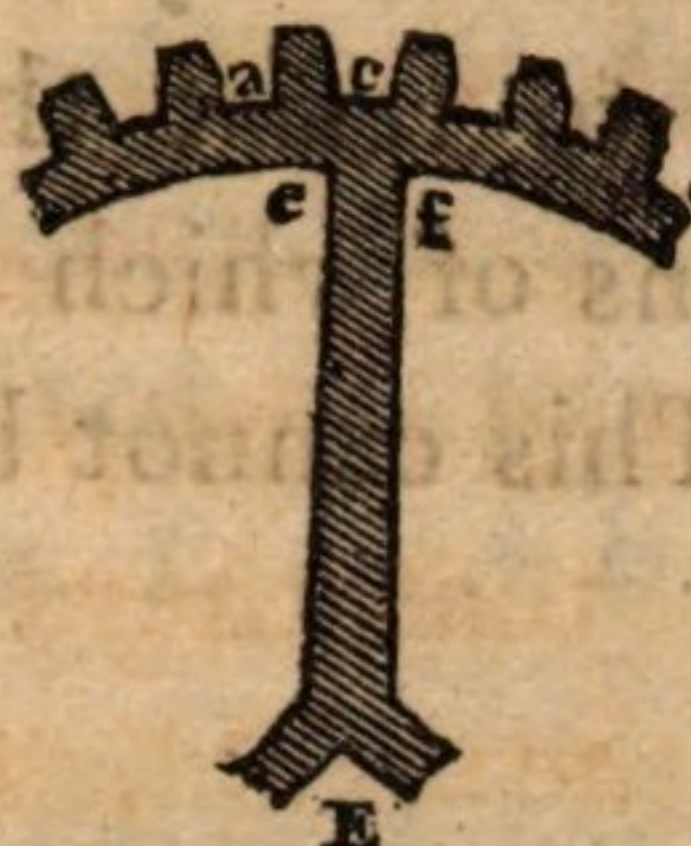


Fig. 2.

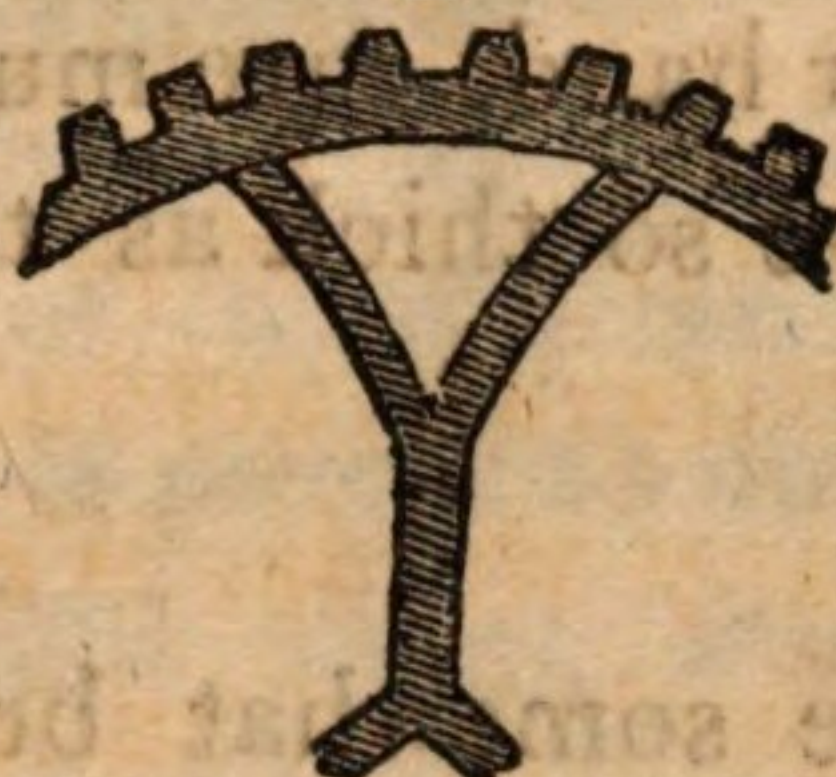


Fig. 3.

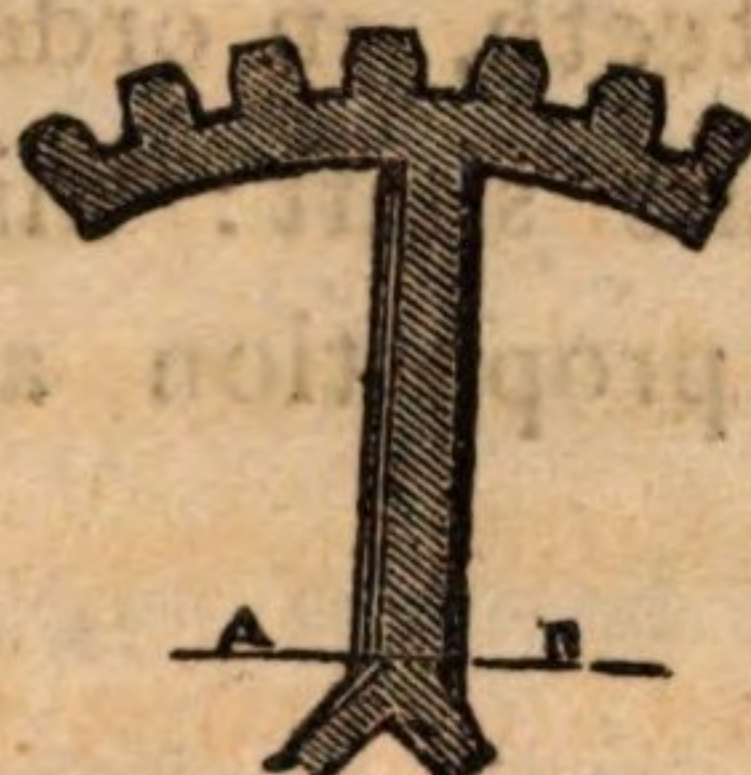


Fig. 4.



let it be so formed as to take off any acute angle which would be apt to break off in sand.

The arms should become larger as they approach the centre of the wheel, (see Emer-

son, page 179) and the eye, E, should be sufficiently strong to resist the driving of the wedges, by means of which it is to be fixed on the shaft. This cannot be brought easily to calculation.

On the other hand, care must be taken not to make the eye so thick as to endanger unequal cooling.

It should be somewhat broader than the breadth of the teeth, in order that it may be the firmer on the shaft: this breadth must be greater in proportion as the wheel is large.

When the ring *a e* is about an inch thick, it is common to make the eye about an inch and quarter thickness, and about $\frac{1}{6}$ broader than the ring, when the wheel is about four feet diameter.

Small wheels have generally but four arms, but it being improper to have a great space of the ring unsupported, the number of arms should be increased in large wheels.

In order to strengthen the arms with little

increase of metal, it is not unusual to make them feathered, which is done by adding a thin plate to the metal at right angles to the arm, as represented by figure third. Fig. 4 is a section of Fig. 3, at A B.

The same rules apply to bevelled wheels; of the practical mode of laying down the working drawings of which we have already spoken. But it is proper to observe, that the eye of a bevelled wheel, should be placed more on that side which is furthest from the centre of the ideal cone of which the wheel forms a part.

When wheels are beyond a certain size, it becomes necessary to have patterns sometimes made for them, cast in parts, which are afterwards united by means of bolts.

A very good mode to prevent the bad effects of unequal contraction, is to have the arms curved, as in the second figure, the curved parts are commonly of the same radius as the wheel, and spring from the half length of the arms.

Materials of Patterns.

The patterns should be made of well seasoned wood. The most proper is clean mahogany, but that being now very expensive, white deal is most commonly used. Beech is very often used for the teeth, and being a close grained wood, it may be made very smooth.

It is almost superfluous to say, that the workmanship of wheel patterns should be such as to produce great accuracy, and a smooth surface, the former being essential to the good movement of the wheels, and the latter to make the patterns produce a good clean impression in the sand.

I am aware, that it is a common practice, in many places, to make teeth very large in the pattern, and after fixing the wheels on their shafts, to chip and file the teeth to the proper size.

I doubt, however, whether this practice be

really advantageous ; for besides the great time which it occupies thus to dress the iron teeth, and the consequent expense, there is the loss of the outer skin (if I may use the expression), of the cast iron, which is by far its most smooth and durable part. In cotton mills, therefore, this method is now but seldom practised *.

As it may be of use to millwrights, I take the liberty of inserting the following tables from a respectable periodical publication.

* Messrs. Peel, Williams, and Co. have, after great time, trouble, and expense, made and arranged a very great number of patterns of wheels, so as to suit almost every case that can in practice occur. They have published a complete list of them, which they intend inserting also in the "Repertory of Arts." In my opinion, what they have done is a material national benefit ; their expense, I am informed, for patterns, has not been less than *four thousand* pounds. There is, however, every reason to think, that it will be an excellent thing ultimately for themselves, as well as of great practical utility to the public.

Table of the Radii of Wheels, from Ten to Three Hundred Teeth, the Pitch being Two Inches. By Mr. B. DONKIN, Millwright, Dartford, Kent.*

No. of Teeth	Radius in Inches.	No.	Radius.	No.	Radius.
10	3,236	34	10,838	58	18,471
11	3,549	35	11,156	59	18,789
12	3,864	36	11,474	60	19,107
13	4,179	37	11,792	61	19,425
14	4,494	38	12,110	62	19,744
15	4,810	39	12,428	63	20,062
16	5,126	40	12,746	64	20,380
17	5,442	41	13,064	65	20,698
18	5,759	42	13,382	66	21,016
19	6,076	43	13,700	67	21,335
20	6,392	44	14,018	68	21,653
21	6,710	45	14,336	69	21,971
22	7,027	46	14,654	70	22,289
23	7,344	47	14,972	71	22,607
24	7,661	48	15,290	72	22,926
25	7,979	49	15,608	73	23,244
26	8,296	50	15,926	74	23,562
27	8,614	51	16,244	75	23,880
28	8,931	52	16,562	76	24,198
29	9,249	53	16,880	77	24,517
30	9,567	54	17,198	78	24,835
31	9,885	55	17,517	79	25,153
32	10,202	56	17,835	80	25,471
33	10,520	57	18,153	81	25,790

* By the Pitch is understood the distance between the centres of two contiguous teeth; and by the radius is understood the distance between the centre of the wheel and the centre of each tooth. For any other pitch, say, as two inches is to the radius in the table, so is the given pitch to the radius required.

No.	Radius.	No.	Radius.	No.	Radius.
82	26,108	118	37,565	154	49,023
83	26,426	119	37,883	155	49,341
84	26,741	120	38,202	156	49,660
85	27,063	121	38,520	157	49,978
86	27,381	122	38,838	158	50,296
87	27,699	123	39,156	159	50,615
88	28,017	124	39,475	160	50,933
89	28,336	125	39,793	161	51,251
90	28,654	126	40,111	162	51,569
91	28,972	127	40,429	163	51,888
92	29,290	128	40,748	164	52,206
93	29,608	129	41,066	165	52,524
94	29,927	130	41,384	166	52,843
95	30,245	131	41,703	167	53,161
96	30,563	132	42,021	168	53,479
97	30,881	133	42,339	169	53,798
98	31,200	134	42,657	170	54,116
99	31,518	135	42,976	171	54,434
100	31,836	136	43,294	172	54,752
101	32,155	137	43,612	173	55,071
102	32,473	138	43,931	174	55,389
103	32,791	139	44,249	175	55,707
104	33,109	140	44,567	176	56,026
105	33,427	141	44,885	177	56,344
106	33,746	142	45,204	178	56,662
107	34,064	143	45,522	179	56,980
108	34,382	144	45,840	180	57,299
109	34,700	145	46,158	181	57,617
110	35,018	146	46,477	182	57,935
111	35,337	147	46,795	183	58,253
112	35,655	148	47,113	184	58,572
113	35,974	149	47,432	185	58,890
114	36,292	150	47,750	186	59,209
115	36,611	151	48,068	187	59,527
116	36,929	152	48,387	188	59,845
117	37,247	153	48,705	189	60,163

164 *An Essay on the Teeth of Wheels.*

No.	Radius.	No.	Radius.	No.	Radius.
190	60,482	227	72,258	264	84,030
191	60,800	228	72,577	265	84,354
192	61,118	229	72,895	266	84,673
193	61,436	230	73,214	267	84,991
194	61,755	231	73,532	268	85,309
195	62,073	232	73,850	269	85,627
196	62,392	233	74,168	270	85,946
197	62,710	234	74,487	271	86,264
198	63,028	235	74,805	272	86,582
199	63,346	236	75,123	273	86,900
200	63,665	237	75,441	274	87,219
201	63,983	238	75,760	275	87,537
202	64,301	239	76,078	276	87,855
203	64,620	240	76,397	277	88,174
204	64,938	241	76,715	278	88,492
205	65,256	242	77,033	279	88,810
206	65,574	243	77,351	280	89,129
207	65,893	244	77,670	281	89,447
208	66,211	245	77,988	282	89,765
209	66,529	246	78,306	283	90,084
210	66,848	247	78,625	284	90,402
211	67,166	248	78,943	285	90,720
212	67,484	249	79,261	286	91,038
213	67,803	250	79,580	287	91,357
214	68,121	251	79,898	288	91,675
215	68,439	252	80,216	289	91,993
216	68,757	253	80,534	290	92,312
217	69,075	254	80,853	291	92,630
218	69,394	255	81,171	292	92,948
219	69,712	256	81,489	293	93,267
220	70,031	257	81,808	294	93,585
221	70,349	258	82,126	295	93,903
222	70,667	259	82,444	296	94,222
223	70,985	260	82,763	297	94,540
224	71,304	261	83,081	298	94,858
225	71,622	262	83,399	299	95,177
226	71,941	263	83,717	300	95,495

THE END.

William Savage, Printer, Bedford Bury.

APPENDIX, No. II.

TO ESSAY ON TEETH OF WHEELS.

ON THE USE OF CHARTS, AND SOME FURTHER
EXPLANATION OF THE CONSTRUCTION OF THE
TABLES OF PITCHES OF WHEEL-WORK.

1. WHEN quantities of any kind, such as time, space, money, &c. expressed in numbers, are mentioned, it often requires a painful exertion of the mind to recollect and compare them. Hence the utility of bringing them into one view in Tables. But there is another mode of comparing quantities not so generally practised, though, in many cases, much more easy and satisfactory to the mind. I allude to charts, in which, instead of using figures, as in tables, the quantities are geometrically represented. This is done by dividing the sides of a square or rectangle into equal parts, and drawing parallel lines at right angles from the divisions. The quantities are pointed off at certain intersections of these scales.

2. When the quantities increase or decrease in arithmetical proportion, as 1, 2, 3, 4, &c. that

proportion will be represented by a straight line, which will pass through these points.

3. But supposing the quantities to increase in geometrical proportion, as 1, 4, 9, 16, &c. the line passing through the points of intersection will form a curve.

These two cases will be best explained by examples.

4. *First*, Suppose the value of any thing to increase as its weight,—the scale on the one side of the square will then represent the value, and that on another the weight.

Let A C Fig. 1st, represent weight (say ounces), and A D value (say shillings). Now, suppose we mark the price of four ounces, it is done by placing a dot opposite to four, on the line of value, and opposite to four on the line of weight, at the intersections of the perpendiculars from these points, which intersection is marked by *b d* on the figure. In the same manner, we may mark the value of 1, 2, 3, 5, 6 ounces. These points are marked *a b c e f*, and the straight line A B passes through them all, and shews the regular progress of the proportion.

It is of no consequence whether the divisions on the line A C be greater or less than

those on A D, provided the lines be divided into equal parts.

5. *Second*, Suppose an accelerating motion, such as that of a falling body, is to be laid down on a chart,—this motion increases as the squares of the times; that is, the body falls a certain distance in the first second of time, four times that distance in the next second, and nine times in the third second, &c.

These points are accordingly marked in figure 2d by *a* opposite to 1 on both scales, by *b* opposite 2 on the scale of time, A D and 4 on that of motion, A C by *c*, opposite 3, on A D, and 9 on A C, &c. the line A B passing through these points forms a curve.

6. When the proportion of any kind is regular, the curve has a regular easy sweep; if otherwise, the curve will undulate, or have irregular windings. Hence, it is a good mode of proving many kinds of tables, to lay down the quantities thus geometrically; for if there be any material error, when the proportion ought to be regular, an elbow will appear in the line A B.

7. Much calculation, too, may often be saved, for when a few of the principle points at some

distance from each other are obtained in the curve, the rest of it may be easily found, by drawing the curve between them with a slip of thin wood, or any other such means of producing an easy curve.

8. Charts, on similar principles, are used for many purposes; for example, there are biographical charts, showing the periods when eminent men appeared, and the relative length of their lives. They are also used for representing revenue of any kind, which generally forms an undulating line, as does also the charts of the heights of the barometer, or the temperature indicated by the thermometer. The heights of mountains, the tides—in short, they may be considered as merely scales of equal parts, and, of course, are applicable to all subjects capable of being represented by numbers.

9. In order to give a more distinct comparative view of the Tables of Pitches, in the “*Essay on the Teeth of Wheels*,” I shall lay their contents down in one chart, but previously, I shall here collect all these tables on one page, and give some further explanation of their mode of construction.

*Reference to Table 1st, in the Essay on the
Teeth of Wheels.*

(10)—Column X is omitted in tables 1st, 3d, and 5th, being only a repetition of the same breadth for all the pitches of each table, but as being perhaps plainer, they are inserted here.

The numbers in column Y are found by squaring the pitch in column W.—(See Proposition I. p. 127; see also p. 138).

Example.

The square of 4, (the pitch in inches) = 16, the value of strength in horse's power.—(See first line of table).

The column Z is found by inverse proposition.—(See Prop. 2d, p. 128.) taking three inches, (the standard pitch), always as the first term, the pitch column, W, as the second, and the horse's power, in column Y, as the third term.

170 *An Essay on the Teeth of Wheels.*

Example.

In. In. Horse's power. Horse's power.

3 : 4 : : 16 : 12 (See first line of table.)

3 : 1 : : 1 : 3 (See last line of table.)

Reference to Table 2d.

(11)—The numbers in column Y are found here by direct proportion from Table 1st.—9, (the breadth in inches in Table 1st), being always the first term of the proportion; the horse's power in Y, table 1st, the second, and the breadth in X, table 2d, the third term.

Example.

In. Horse's power. In. Horse's power.

9 : 16 : : 8 14.22 (See table 2d, line first.)

Column Z is found, as in table 1st, by inverse proportion, 3 inches, (the standard pitch), being always the first term. Thus,

In. In. Horse's power. Horse's power.

3 : 4 : : 14.22 : 10.66 See 1st line of table.

Reference to Table 3d.

(12)—The numbers in column Y are found by direct proportion, taking 9, (the square of the

standard pitch of three inches), as the first term, and the square of the pitch in W as the 2d term.

Example.

As 9 (the square of 3 inch pitch)

Is to 16 (the square of 4 inch pitch),

So is 46 horses power (the value in column Y of 3 inch pitch,)

To 81.77—(See first line of table).

Column Z is found by inverse proportion, as in former tables.

Example.

In. In. Horse's power. Horse's power.
3 : 4 : : 81.77 : 61.33.—(See 1st line of table).

Reference to Table 4th.

(13)—The numbers in column Y are found here in a manner similar to table 2d, by direct proportion.

Example.

In. In. Horse's power. Horse's power.
8 : 7 : : 62.61 : 54.78.—(See 2d line of table).

172 *An Essay on the Teeth of Wheels.*

The numbers in column Z are found, as in the former tables, by inverse proportion,

In. In. Horse's power. Horse's power.
 $3 : 4 :: 81.77 : 61.33$.—(See 1st line of table).

Reference to Table 5th.

(14)—The numbers in column Y of this table are found by direct proportion from column Y of table 4th, 11 feet (velocity per second), being always the first term, and 3 feet (velocity), the second term

Ft. Ft. Horse's power. Horse's power.
 Thus, $11 : 3 :: 81.77 = 22.30$.—(See 1st line of table).

Column Z is found by inverse proportion, as in all the former tables :

In. In. Horse's power. Horse's power.
 Thus, $3 : 4 :: 22.30 : 16.72$.—(See 1st line of tables).

Reference to Table 6th.

(15)—The numbers in column Y and Z, in this table, are found from table 5th, in the same manner as those columns in table 2d are formed from table 1st. The only difference in table 6th from table 5th, is that which arises from the difference of the *breadth* of the teeth.

Explanation of the Chart.

16. The scale on the line A B represents the pitch in inches.

The scale on the line A C the horse's power ; a single example will probably be sufficient to illustrate the use of the chart.

Suppose the pitch to be $3\frac{1}{2}$ inches, let it be required to find the horse's power to which that pitch is equal when moving at 3 feet per second, in situations where properly greased and free from sand—observe, where the line from the pitch $3\frac{1}{2}$ intersects the curve Z of Table VI, perpendicular to the point of intersection, on the line A C, will be found 12.79 on the scale or the horse's power to which $3\frac{1}{2}$ pitch, when the teeth are 7 inches broad, is equal, after making allowance for the length of the teeth. (See Table VI, also Essay on Teeth of Wheels, p. 139).

Observations.

17.—1st, It will be observed, that the curves Y and Z intersect each other, for all the tables on the pitch line marked 3 inches, because that is the standard. (See 1st Essay, p. 137).

18.—2d, The curves, continued from 1 inch pitch downward, unite in the points marked *O*, being the commencement of the scale of pitches, and this part of the curve gives the horse's power equal to any fraction of an inch.

19.—3d, It was already observed, p. 139, that durability as well as strength, should be considered in this investigation. The true proportion, therefore, may be somewhere between the curves *Y* and *Z*, but nearer to *Z* than *Y*. For although long teeth will be more easily broken than short ones, yet while they do not break, the strain being generally diffused over a greater number of teeth, they will wear longer.

20.—4th, The pitch is laid down on *AB*, real measure, so that if the pitch should happen to be fractional, it may be taken by a pair of compasses and applied to the chart, which will at once indicate the power to which it may be equal at certain velocities.

I HOPE the following letter, respecting the strength, &c. of wheel-work, for which I am indebted to Mr JAMES CARMICHAEL, millwright, (now of Dundee,) will not be unacceptable to the reader :

“ SIR—It is a corroboration of the truth of the tables of pitches, that Mr Robertson’s table coincides very nearly with columns marked Y in your tables ; but he seems to have overlooked the propriety of taking the length of the teeth into his calculations. I am, therefore, still of opinion, that the true value is in the columns marked Z in your tables.

“ Admitting the truth of the fundamental propositions, and, from a comparison of the tables of pitches, I would propose the following rule, which is on the same principle as that of columns Z in your tables, for calculating the proportionate strength of the teeth of wheels.

Rule.

“ Multiply the breadth of the teeth by the square of the thickness, and divide the product by the length. The quotient will be the proportionate strength in horse’s power, with a velocity of 2·27 feet per second.

“ By that rule I have calculated the following table ; and, for the sake of comparison, I have taken three cases from Mr Robertson’s table, and three from your tables 3d and 5th.

Explanation of the Table.

“ Column 2d contains the thickness of the teeth. The pitch is found by multiplying the thickness by 2·1 *; and the length is found by multiplying the thickness by 1·2 †.

“ Column 5th contains the proportionate strength, and also the number of horse’s power (proportionate to the case H, *see page 143*), which the teeth is equal to, with a velocity of 2·27 feet per second.

1	2	3	4	5	6	7	8
Pitch in inches.	Thick-ness of teeth in inches.	Breadth of teeth in inch.	Length of teeth in inch.	Strength of teeth, or number of horse’s power, at 2·27 feet persecond.	Horse’s power, at three feet per second.	Horse’s power, at six feet per second.	Horse’s power, at 11 feet per second.
3·9	1·9	7·6	2·28	11·73	15·46	30·92	56·84
2·9	1·4	5·6	1·68	6·53	8·63	17·26	31·64
2·1	1·	4·	1·2	3·33	4·4	8·8	16·1
4·	1·904	8·	2·285	12·698	16·78	33·56	61·52
3·	1·428	8·	1·714	9·523	12·58	25·16	46·
1½·	·714	8·	·857	4·752	6·27	12·54	23·02

* That is in order to make the space between the teeth a little wider than the thickness of a tooth. See p. 151.

† Respecting the length of teeth, See p. 153—155.

Remarks.

“ 1st, The three last cases in the table are taken from the tables 3d and 5th, and the results coincide so well with the columns marked Z, that I presume the rule is just.

“ 2dly, The three first cases are from Mr Robertson’s table ; the second case is very near the same as in Mr R.’s table ; but the first is considerably less, and the third considerably more. Hence I infer that Mr R. has taken his data from a pitch about three inches.

“ 3dly, If any two wheels have the length and thickness of their teeth in the same proportion to their respective pitches, the breadth of the teeth and the velocity being the same, *the strength will be directly as the pitches.* The truth of this is deduced from the columns marked Z in tables 1st, 3d, and 5th.

“ I am,

“ SIR,

“ Your most obedient servant,

“ JAMES CARMICHAEL.

“ GLASGOW, }
December, 1808.” }

TABLE OF PITCHES OF WHEEL- WORK,

With the *breadth* and *thickness* of the teeth,
and the corresponding number of horse's
power, calculated by Mr CARMICHAEL'S
rule.

Pitch in inches.	Thick- ness of teeth in inches.	Breadth of teeth in inch.	Length of teeth in inch.	Strength of teeth, or num- ber of horse's power, at 2·27 feet per second.	Horse's power, at three feet per se- cond.	Horse's power, at 6 feet per se- cond.	Horse's power, at 11 feet per second.
3·99	1·9	7·6	2·28	12·03	15·90	31·80	58·30
3·78	1·8	7·2	2·16	10·80	14·27	28·54	52·32
3·57	1·7	6·8	2·04	9·63	12·72	25·54	46·68
3·36	1·6	6·4	1·92	8·53	11·27	22·54	41·32
3·15	1·5	6·0	1·80	7·50	9·91	19·82	36·33
2·94	1·4	5·6	1·68	6·53	8·63	17·26	31·64
2·73	1·3	5·2	1·56	5·63	7·44	14·88	27·28
2·52	1·2	4·8	1·44	4·80	6·34	12·68	23·24
2·31	1·1	4·4	1·32	4·03	5·32	10·64	19·54
2·10	1·0	4·0	1·20	3·33	4·40	8·81	16·15
1·89	0·9	3·6	1·08	2·70	3·57	7·14	13·09
1·68	0·8	3·2	0·96	2·13	2·81	5·62	10·33
1·47	0·7	2·8	0·84	1·63	2·15	4·30	7·88
1·26	0·6	2·4	0·72	1·20	1·59	3·18	5·83
1·05	0·5	2·0	0·60	0·83	1·10	2·20	4·03

ERRATA

OF THE ESSAY ON THE TEETH OF WHEELS.

- Page 41, line 1, *for vious, read obvious,*
 — 129, line 1, *for the strength is, read the stress is,*
 — 137, line 8, *for B and F, read B and D,*
 — 139, line 21, *for description of the table of pitches, read de-*
 scription of the six following tables of pitches,
 — 143, line 1, *for the strength, read the stress,*
 — 144, table III, line 2, *for 46·95, read 53·66,*
 — do. — do. line 3, *for 34·5, read 46,*
 — do. — do. line 4, *for 23·54, read 38·33,*
 — do. — do. line 5, *for 15·23, read 30·66,*
 — do. — do. line 6, *for 8·62, read 23,*
 — do. — do. line 7, *for 3·84, read 15·33,*
 — 145, table IV, line 4, *for 23·55 read 23·94,*
 — do. — do. line 5, *for 22 read 10·22,*
 — 146, table V, line 2, *for 12·79, read 14·63,*
 — do. — do. line 3, *for 9·40, read 12·54,*
 — do. — do. line 4, *for 6·53, read 10·45,*
 — do. — do. line 5, *for 4·17, read 8·36,*
 — do. — do. line 6, *for 2·34, read 6·26,*
 — do. — do. line 7, *for 1·02, read 4·17,*
 — 147, table VI, line 4, *for 3·44, read 5·44,*
 — 152, — do. line 7, *for 17·84, read 17·54.*

15 90
 15 50
 ———
 20

IN MILLWORK.

Name.	Remarks.
Water Wheel . . .	Has been 16 years at work—Teeth much worn.
Water Wheel . . .	{ Has been 16 years at work. This geering was found rather too narrow for the strain, as it is wearing much faster than the rest of the wheels in the same mill.
Water Wheel . . .	
Water Wheel . . .	{ The only defect in this geering, which has been 16 years at work, is the want of breadth in the spur wheel and pinion; they ought to have been 6 inches or more, as they will not last half so long as the bevel wheels and pinions connected with them.
Horse Mill . . .	
Horse Mill . . .	
Steam Engine, by B. a	
Steam Engine, by B. a	
Steam Engine by B. a	
Steam Engine, by B. a	
Steam Engine . . .	
Ditto Ditto . . .	This is better pitch for the power than Q.
Ditto Ditto . . .	{ This wheel has wooden teeth, and has been working for 3 years past.
Ditto Ditto . . .	
Ditto Ditto . . .	Ditto Ditto.
Ditto Ditto . . .	
Ditto Ditto . . .	This pitch has been found to be too fine.
Ditto Ditto . . .	

TABLE OF PRICES OF

Name	No. of Pounds	No. of Cents	No. of Mills	Price	
				Per Cwt.	Per Lb.
White Sugar	11	11	11	11	11
Yellow Sugar	11	11	11	11	11
Green Sugar	11	11	11	11	11
White Coffee	11	11	11	11	11
Yellow Coffee	11	11	11	11	11
Green Coffee	11	11	11	11	11
White Tea	11	11	11	11	11
Yellow Tea	11	11	11	11	11
Green Tea	11	11	11	11	11
White Rice	11	11	11	11	11
Yellow Rice	11	11	11	11	11
Green Rice	11	11	11	11	11
White Beans	11	11	11	11	11
Yellow Beans	11	11	11	11	11
Green Beans	11	11	11	11	11
White Corn	11	11	11	11	11
Yellow Corn	11	11	11	11	11
Green Corn	11	11	11	11	11
White Potatoes	11	11	11	11	11
Yellow Potatoes	11	11	11	11	11
Green Potatoes	11	11	11	11	11
White Apples	11	11	11	11	11
Yellow Apples	11	11	11	11	11
Green Apples	11	11	11	11	11
White Peaches	11	11	11	11	11
Yellow Peaches	11	11	11	11	11
Green Peaches	11	11	11	11	11
White Plums	11	11	11	11	11
Yellow Plums	11	11	11	11	11
Green Plums	11	11	11	11	11
White Cherries	11	11	11	11	11
Yellow Cherries	11	11	11	11	11
Green Cherries	11	11	11	11	11
White Strawberries	11	11	11	11	11
Yellow Strawberries	11	11	11	11	11
Green Strawberries	11	11	11	11	11
White Raspberries	11	11	11	11	11
Yellow Raspberries	11	11	11	11	11
Green Raspberries	11	11	11	11	11
White Blackberries	11	11	11	11	11
Yellow Blackberries	11	11	11	11	11
Green Blackberries	11	11	11	11	11
White Blueberries	11	11	11	11	11
Yellow Blueberries	11	11	11	11	11
Green Blueberries	11	11	11	11	11
White Elderberries	11	11	11	11	11
Yellow Elderberries	11	11	11	11	11
Green Elderberries	11	11	11	11	11
White Currants	11	11	11	11	11
Yellow Currants	11	11	11	11	11
Green Currants	11	11	11	11	11
White Grapes	11	11	11	11	11
Yellow Grapes	11	11	11	11	11
Green Grapes	11	11	11	11	11
White Pears	11	11	11	11	11
Yellow Pears	11	11	11	11	11
Green Pears	11	11	11	11	11
White Quinces	11	11	11	11	11
Yellow Quinces	11	11	11	11	11
Green Quinces	11	11	11	11	11
White Apples	11	11	11	11	11
Yellow Apples	11	11	11	11	11
Green Apples	11	11	11	11	11
White Peaches	11	11	11	11	11
Yellow Peaches	11	11	11	11	11
Green Peaches	11	11	11	11	11
White Plums	11	11	11	11	11
Yellow Plums	11	11	11	11	11
Green Plums	11	11	11	11	11
White Cherries	11	11	11	11	11
Yellow Cherries	11	11	11	11	11
Green Cherries	11	11	11	11	11
White Strawberries	11	11	11	11	11
Yellow Strawberries	11	11	11	11	11
Green Strawberries	11	11	11	11	11
White Raspberries	11	11	11	11	11
Yellow Raspberries	11	11	11	11	11
Green Raspberries	11	11	11	11	11
White Blackberries	11	11	11	11	11
Yellow Blackberries	11	11	11	11	11
Green Blackberries	11	11	11	11	11
White Blueberries	11	11	11	11	11
Yellow Blueberries	11	11	11	11	11
Green Blueberries	11	11	11	11	11
White Elderberries	11	11	11	11	11
Yellow Elderberries	11	11	11	11	11
Green Elderberries	11	11	11	11	11
White Currants	11	11	11	11	11
Yellow Currants	11	11	11	11	11
Green Currants	11	11	11	11	11
White Grapes	11	11	11	11	11
Yellow Grapes	11	11	11	11	11
Green Grapes	11	11	11	11	11
White Pears	11	11	11	11	11
Yellow Pears	11	11	11	11	11
Green Pears	11	11	11	11	11
White Quinces	11	11	11	11	11
Yellow Quinces	11	11	11	11	11
Green Quinces	11	11	11	11	11

TABLES OF PITCHES OF WHEEL WORK.

(See APPENDIX to ESSAY on TEETH of WHEELS, p. 121—149.)

TABLE I.

Velocity of the Pitch Line being 3 Feet per Second, and
The Velocity being 3 Feet per Second, and breadth of Teeth 9 Inches.

W	X	Y	Z
Pitch in inches.	Breadth of teeth in inches.	Value of strength in horse's power.	Value of strength in horse's power.
4	8	22.30	16.72
3½	8	17.07	14.63
3	8	12.54	12.54
2½	8	8.71	10.45
2	8	5.57	8.36
1½	8	3.13	6.26
1	8	1.39	4.17

TABLE II.

The Velocity being 3 Feet per Second, and the breadth of the
The Velocity being 3 Feet per Second, and breadth double each Pitch.

W	X	Y	Z
Pitch in inches.	Twice the pitch in breadth.	Value of strength in horse's power.	Value of strength in horse's power.
4	8	22.30	16.72
3½	7	14.93	12.79
3	6	9.4	9.4
2½	5	5.44	6.53
2	4	2.78	4.17
1½	3	1.17	2.34
1	2	0.34	1.02

THE ALPHABETICALLY ORDERED

THE ALPHABETICALLY ORDERED

TABLE IV

TABLE III

Y	X	Z	W	V	U
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00

Y	X	Z	W	V	U
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00

TABLE IV

TABLE III

TABLE IV

TABLE III

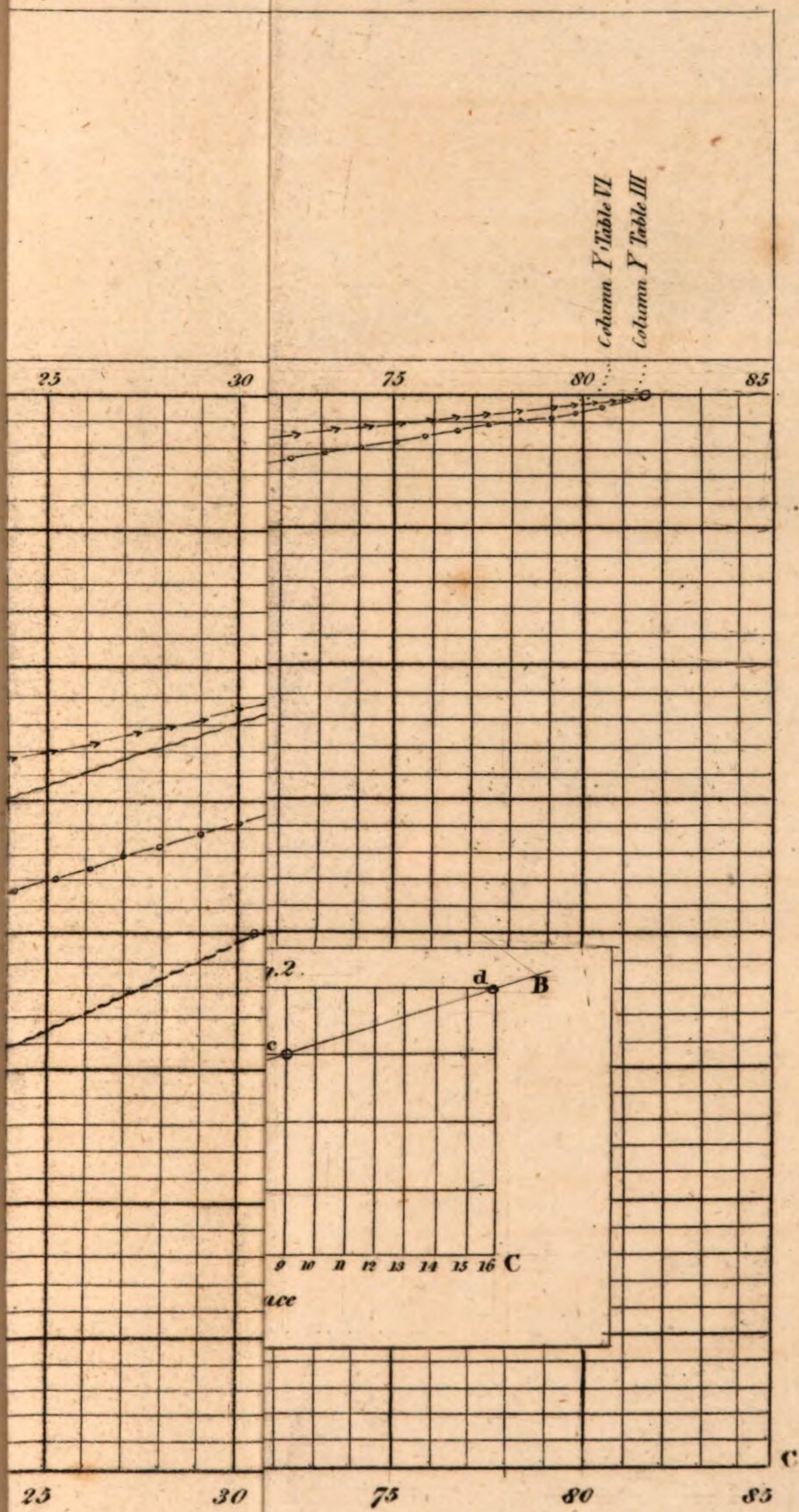
TABLE IV

TABLE III

TABLE IV

TABLE IV

ART Showing the Horse



I N D E X

TO

ESSAY I.

ON THE

TEETH OF WHEELS.

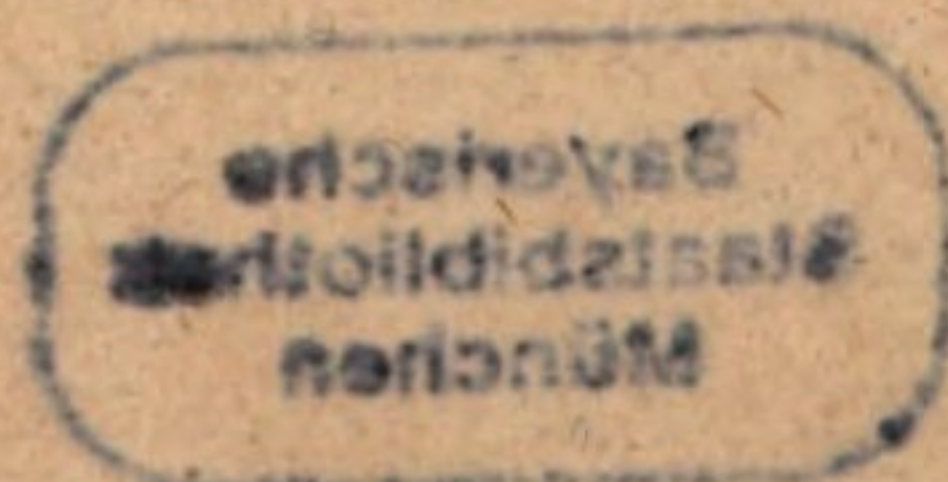
	PAGE		PAGE
ANGLE, definition of an	111	Conducted wheel	53—71
Arch, keystone of an	53	Conducting wheel	52—68
Arc of a circle, definition		Conductors -	54—73
of the -	112	Cones 81, 83, 84, 85, 86, 159	
		——, definition of a	116
Beech, used for patterns	160	——, proportional	88
Bevel gear 7, 81, 84, 85, 87		Corollary, term	117
—— wheels 78, 84, 159		Cycloid, form of teeth 78, 79,	
Boulton and Watt, Messrs,		109	
126, 143		Cylinder, definition of a	116
Breadth of teeth 151, 152		Cylindric staves, term for	
Brewster, Dr -	96	teeth 37, 44, 45, 56, 106	
Camus on the teeth of		Desagulier's calculations	
wheels, &c. 6, 9, 26, 53, 97		130—134	
Cast iron pinions	126	Diameter of a circle, de-	
——— trundle -	60	finition of the	112
——— wheels 59, 91		Donkin, Mr B.'s tables	162
Chord of an arc, definition		Du Buat - -	138
of the -	113	Emerson's mechanics 120,	
Circle, definition of a	111	131, 158	
——, exterior -	77	Encyclopædia Britannica	96
——, interior -	77	Epicycloid -	19—108
Cogs, term for teeth	12		

182 *Index to Essay on Teeth of Wheels.*

	PAGE		PAGE
Epicycloid, exterior	21—30	Interior Epicycloid	22, 27, 28,
———, interior	22, 27, 28,		74
	74	Internal pinion	7, 74, 76, 77
———, spherical	86, 101—19	Involute of a circle	96, 107,
Exterior circle	- 77		109, 110
——— epicycloid	21—30, 96	Johnson, Dr	- 78
External pinion	- 74	Journals	- - 123
——— spur wheels	78	Journeys	- - 123
Face wheel	- 84	Key-stone of an arch	53
Fenton, Murray and		Lahire, De	6, 96, 97, 109
Wood, Messrs	24	Lantern	- - 11
Ferguson's lectures	- 96	Leaves, term for teeth	12—80
Flanks of teeth	49—50	Lesslie, Professor	118
Forms of teeth	- 101	Line of centres	13—153
Friction	- 50—149	Materials of patterns	160
——— of wheelwork	101—109	Metal, subject to pressure	125
Geering, mode of	154	——— teeth	- 52
General observations on		Microscope	- 51
the wheel-work of mills	122	Murray, Mr, of Leeds	71, 73
Generating circle	21—86	Overshot water wheels	135
Horse mill	- 136, 154	Parallel lines, definition of	114
Horse's power, steam en-		Patterns, materials of	160
gines distinguished by	130,	———, mode of making	91
	135, 136	——— of cast - iron	
———, term	129—152	wheels	- 153, 155
Hutton on clock-work	154	Peel, Williams and Co.,	
Impelling teeth	- 105	Messrs	- 161
Inequalities of the surfaces			
of teeth	50, 51, 52		
Interior circle	- 77		

Index to Essay on Teeth of Wheels. 183

	PAGE		PAGE
Performance of men by		Spherical epicycloid	86, 87
machines -	134	Spur gear, 7, 36, 78, 84, 85, 87	
Perpendicular, term, 114, 115,		— wheels -	8, 92
116		— — — external	78
Pinion -	11—136	Staves, term for teeth	12—106
—, external	74, 76, 77	Steam engines, distin-	
—, internal	7, 74, 76, 77	guished by horse's	
Pinions of cast-iron	126	power	130, 135, 136
Pitch line -	14—162	Strain -	129, 147
Polygon, definition of a	113	Strength of men	133, 134
Power, term -	117	— of teeth	151
Principal diameter	88	— of timber	127
Proportional circles	14—74	Stress -	143, 149
— cone	88		
— diameters	87—88	Table of pitches of wheels	
— pinion	47	136—152	
— radii	14, 19, 32, 47	— of the radii of	
Proposition, term	117	wheels	162, 163, 164
		— of wheels in actual	
Rack and pinion	7, 78, 80	use in millwork	135, 144
Radius of a circle, defini-		Tangent of a circle, defi-	
tion of the -	112	nition of the -	113
Real radii	14, 102, 104	Teeth -	12—139
Rees' Cyclopedia	96	—, breadth of	151, 152
Remarks on the friction of		—, inequalities of their	
wheel work	101—109	surfaces	50, 51, 52
Roberton, Mr John, engi-		—, metal -	52
neer, - -	150	—, wooden	50, 51
Robison, Professor	92—96	—, strength of	151
Rule to determine the length		— thickness of	150, 151,
of the teeth of wheels,	72	152	
Rupp, Mr, of Manchester	71	Thread, manner of lap-	
		ping -	93, 94
Smeaton, Mr,	117, 131, 134	Timber, strength of	127



184 *Index to Essay on Teeth of Wheels.*

	PAGE		PAGE.
Timber, subject to pres-		Wheel and pinion	7, 45, 48
sure - - -	125	88, 123	
Torsion - - -	123	—— and trundle	7, 37, 41
Triangle, definition of a	111	—— bevelled	- 78
Trundle - - -	11—123	—— cast-iron	59, 91
——, cast-iron - -	60	—— having cast-iron	
Twist - - -	123	teeth - - -	129
		—— work, construc-	
Velocity, term - -	117	tion of - - -	50, 58
		Wooden teeth - -	51
Wallower - - -	11	Young, Dr, natural philo-	
Water-wheel	136, 137	sophy 98, 100, 101, 119,	
Watt, Mr - - -	66, 131	132	
Wheel - - -	11—105		

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$$1/29$$

$$\begin{array}{r} 9 \\ 2 \\ \hline 18 \end{array}$$

2.

Buchanan, Robertson

An essay on the teeth of wheels comprehending principles, and their application in practice, to millwork and other machinery ; with numerous figures

London 1808

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